

Design of Robust PID Controllers Using H_∞ Technique to Control the Frequency of Wind-Diesel-Hydro Hybrid Power System

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ABSTRACT

This paper proposes and provides the design steps of three robust output feedback controllers to control the frequency of Wind-Diesel-Hydro hybrid system. The first presents a centralized robust based H_∞ (CRH $_\infty$) controller. The role of H_∞ is to minimize the disturbance effect on the system output. The effect of the LMI tuning variables of RH $_\infty$ controller on the system dynamic performance is presented and discussed. The controllers are solved using the Linear Matrix Inequalities (LMI) technique and characterized by a similar size as the plant that may be of higher order and thus creates difficulty in implementation in large systems. The second presents decentralized robust based H_∞ for each unit (DRH $_\infty$). The third is robust PID controllers which are ideally practical for industry and more appealing from an implementation point of view since its size is lower. The optimum parameters of the robust PID controllers are found through the optimization by a novel combination of RH $_\infty$ control theories through the Genetic Algorithm (GA) technique. More specifically, the third robust PID controllers are proposed to achieve the same robust performance as decentralized (DRH $_\infty$) controllers, respectively. All controllers are used as load frequency controllers to control the Wind-Diesel-Hydro hybrid system. Comparisons of the performance of the three robust output feedback controllers under diverse tests in different disturbances and variation in the plant parameters are carried out.

Keywords : H_∞ control, Load Frequency Control, linear matrix inequalities (LMI), Robust PID, Hybrid System

1. INTRODUCTION

The hybrid wind-diesel power system is considered economically for supply of electrical energy to remote and isolated areas (hilly areas and islands) where the wind speed is considerable for electrical generation and electric energy is not easily available from the grid.

To meet the increasing load demand for an isolated community, expansion of this hybrid power system is required. Hydro generating unit is added in parallel where water streams are abundantly available. The resulting Wind diesel hydro hybrid power system must provide good quality service to the consumer load, which depends mostly on the type and action of the generation controller[1].

In a power system, load-frequency control (LFC) plays an essential role to allow power exchanges and to supply better conditions for the electricity trading. Load frequency control in power systems is very important in order to supply reliable electric power with good quality. The goal of the LFC is to maintain zero steady state errors in a multi area interconnected power system. The PID controller has been widely used in load frequency. Due to its functional simplicity and performance robustness, Designing and tuning of PID controllers have been a large research area ever since Ziegler and Nichols presented their methods in 1942 [2]. Specifications, stability, design, applications and performance of the PID controller have been widely treated since then ([3], [4]).

Robust controllers based on the optimization of the H_{∞} -norm of the transfer matrix between the system disturbance and its output, via Riccati method or Linear Matrix Inequalities (LMI) technique [5-8] have been widely applied in control theory and applications. Such controllers show robustness against disturbance but may have a large size that may give rise to complex structure and creates difficulty in implementation.

To overcome this difficulty, one has to reduce the size of controller for a high order plant by various reduction algorithms which have been proposed in [12]. Others, as a variation, use a specific controller structure (e.g. lead lag or PI/PID Proportional-Integral and derivative), whose parameters can be determined via the minimization of the system robust norm using a different optimization technique [11,18-20] or an iterative LMI technique [21-23]. There is thus a need for a controller that achieves the same robust performance as simplicity in design and implementation.

This paper proposes and provides the design steps of three robust controllers. The first controller CRH_{∞} , which are solved using the linear matrix inequalities technique and results in very high order controller. The effects of the tuning variables of CRH_{∞} controller on the system dynamic performance are given and discussed. In the considered hybrid system application, the role of H_{∞} is to minimize the load disturbance effect on the output frequency represented by the deviation in the change in frequency. The third is robust PID controllers which have a simpler structure and more appealing from an implementation point of view. The parameters of the robust PID controllers are optimized by novel combinations of RH_{∞} control theories through GA. The cost functions (energy) to be minimized via GA are represented by RH_{∞} norms. The optimization objectives are used to tune the parameters of the PID controllers for achieving the same robust performance as DRH_{∞} controllers. The third controller is named PID/ H_{∞} . The proposed robust controllers are applied to a wind-diesel-hydro hybrid system. The designed robust PID controllers are compared with DRH_{∞} and CRH controllers when the system is subjected to a severe disturbance with different

operating conditions. The results show that the Decentralized PID/ H_∞ controllers guarantee the robust performance as well as the DRH $_\infty$ and CRH $_\infty$ controllers.

2. HYBRID POWER SYSTEM MODELING

In this study, an isolated wind-diesel-hydro hybrid power system is chosen and load frequency control of this system is made first by **Centralized H**, then by **Decentralized H** and finally by **Decentralized PID/ H_∞** . In the hybrid system considered, synchronous generator is connected on diesel-side and induction generator is connected on wind side and hydro system is added in parallel.

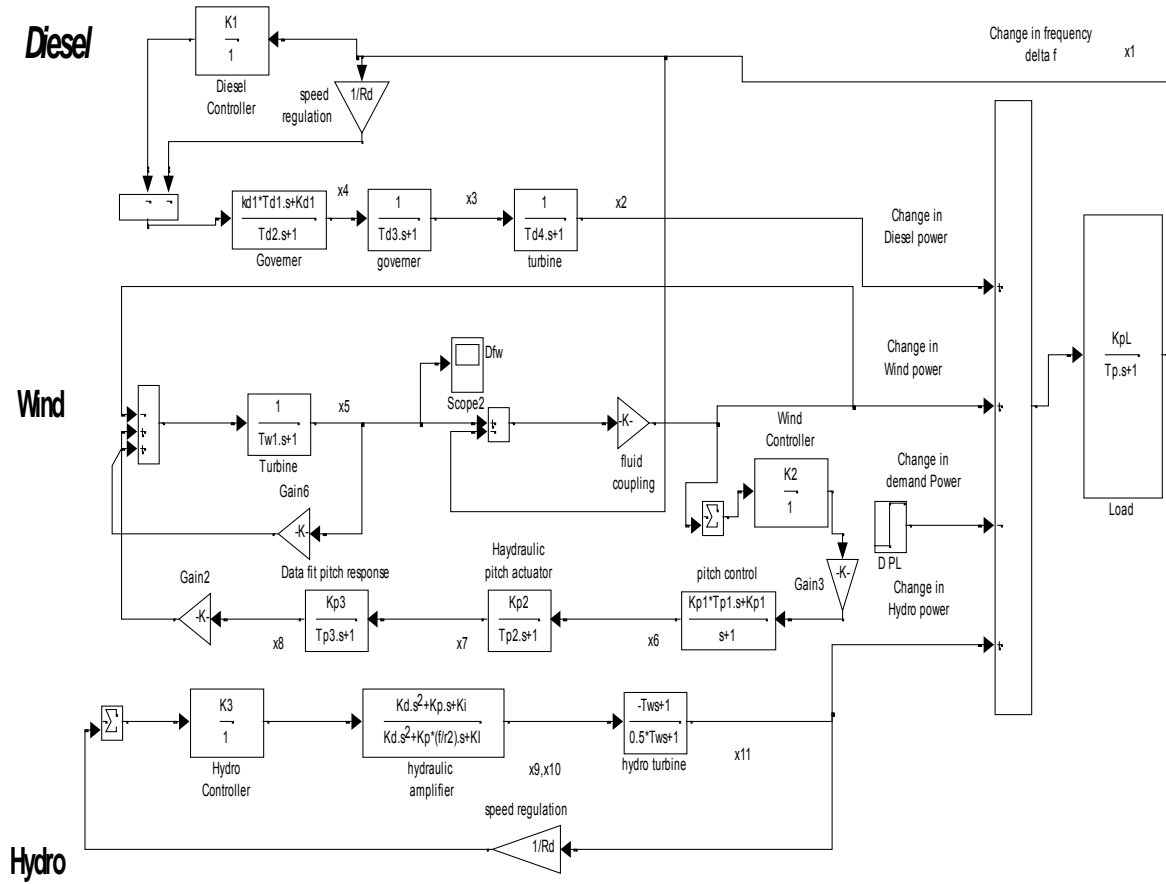


Fig. 1: Simulink model of the hybrid power system

The state equations of the sample power system can be written in the vector-matrix differential equation form as:

$$\dot{x} = Ax + Bu \quad (1)$$

where x is the state vector, $x_1 = f$. where f is the change in system frequency.

u is the control vector

$$u = [-P_L]^t$$

and A and B are matrices and depend on the loading conditions and excitation level.

The system matrices, system variable definition and parameter values (Tables 1 and 2) are defined in Appendix .

3. ROBUST H_∞ CONTROLLER (RH_∞)

In a typical H_∞ design problem, the nominal plant model represented by its transfer function $G(s)$ is usually known and the design problem for an output feedback control is formulated as a standard H_∞ problem, as described by the block diagram of Fig. 2. $P(s)$ and $K(s)$ represent the plant and the controller transfer functions in Laplace domain respectively. The controller is aimed to be designed using the H_∞ design technique. In the block diagram, w represents the external disturbances, z the regulated outputs and y the measured outputs. The vector u consists of the controlled inputs. Let:

$$P(s): \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} u \end{cases} \quad (2)$$

$$\text{Controller: } K(s): \begin{cases} \dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K + D_K y \end{cases} \quad (3)$$

be state-space realizations of the plant $P(s)$ and controller $K(s)$, respectively, and let

$$\begin{cases} \dot{x}_{CL} = A_{CL} x_{CL} + B_{CL} w \\ z = C_{CL} x_{CL} + D_{CL} w \end{cases} \quad (4)$$

be the corresponding closed-loop state-space equations with $x_{CL} = [x \quad x_K]^T$

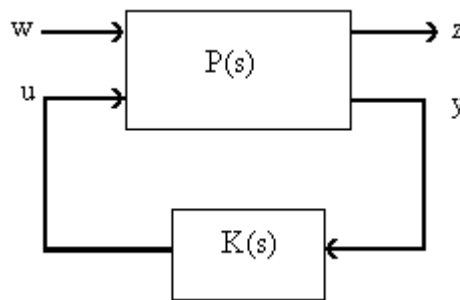


Fig. 2 Output feedback block diagram

The design objective for finding $K(s)$ is to optimize the H_∞ -norm of the closed-loop transfer $G(s)$ from w to z , i.e.,

$$G(s) = C_{CL} (s - A_{CL})^{-1} B_{CL} + D_{CL} \quad (5)$$

And

$$|G(s)_{ZW}| <$$

using the LMI technique. γ is a specific number. This can be fulfilled if and only if there

exists a symmetric matrix X such that the following LMIs are satisfied.

$$\begin{pmatrix} A_{CL}X + XA_{CL}^T & B_{CL} & XC_{CL}^T \\ B_{CL}^T & -I & D_{CL}^T \\ C_{CL}X & D_{CL} & -2I \end{pmatrix} < 0 \quad (6)$$

$X > 0$

Equation (6) represents the system disturbance rejection, i.e., minimization of the effect of the worst-case disturbance on the output. LMI toolbox can be used for such controller design [6,13]. Where;

$$A_{CL} = \begin{bmatrix} A + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix} \quad B_{CL} = \begin{bmatrix} B_1 + B_2 D_K D_{21} \\ B_K D_{21} \end{bmatrix}$$

$$C_{CL} = [(C_1 + D_{12} D_K C_2) \quad D_{12} C_K]$$

$$D_{CL} = [D_{11} + D_{12} D_K D_{21}]$$

LMI constraints defined by (6) can be derived from:

- Stability condition based on Lyapunov energy function;

$$V(x) = x^T X x > 0 \quad (7)$$

$$dV/dt = x^T (A^T X + X A) x + x^T (X B) u + u^T (B^T X) x < 0 \quad (8)$$

From equation (8) stability LMI constraints is;

$$\begin{pmatrix} A_{CL}^T X + X A_{CL} & X B_{CL} \\ B_{CL}^T & -2I \end{pmatrix} < 0 \quad (9)$$

$X > 0$

- Minimization of the disturbance effect condition on the selected outputs based on infinity norm (H_∞) that equal; $y^T y - \gamma^2 u^T u < 0$ (10)

From equation (10) the disturbance effect under LMI constraints is;

$$\begin{pmatrix} C_{CL}^T C_{CL} & C_{CL}^T D_{CL} \\ D_{CL}^T C_{CL} & D_{CL}^T D_{CL} \end{pmatrix} < 0 \quad (11)$$

From equations (9) and (11) LMI constraints become;

$$\begin{pmatrix} A_{CL}^T X + X A_{CL} + C_{CL}^T C_{CL} & X B_{CL} + C_{CL}^T D_{CL} \\ B_{CL}^T X + D_{CL}^T C_{CL} & D_{CL}^T D_{CL} - 2I \end{pmatrix} < 0 \quad (12)$$

$$X = X^T > 0 \quad (\text{Positive definite matrix})$$

According to the Schur complement LMI constraints defined by (12) become as given in (6) [16, 17].

The steps of designing robust H_∞ Output-Feedback using LMI toolbox can be summarized as follows:

Step 1: Form the plant (power system) as a Matlab system:

$A=A; B=[B_1;B_2]; C=[C_1;C_2]; D=[D_{11};D_{12};D_{21}];$
 $P = \text{Itisys} (A,B,C,D) : P$ system plant

Step 2 : Determine the H_∞ controller $K(s)$ with: assumed given γ

$[\gamma_{\text{opt}}, K] = \text{hinflmi} (P, [1 \ 1], \gamma)$
 with 1-input 1-output

Step 3: Construct the closed loop system

$\text{clsys} = \text{slft} (P, K, 1, 1)$

Step 4: Extract closed-loop state-space matrices

$[a, b_1, b_2, c_1, c_2, d_{11}, d_{12}, d_{21}, d_{22}]$
 $= \text{hinfpar}(\text{clsys}, [1 \ 1]); A_{\text{cl}} = a, B_{\text{cl}} = [b_1 \ b_2], C_{\text{cl}} = [c_1; c_2], D_{\text{cl}} = [d_{11} \ d_{12}; d_{21} \ d_{22}]$

Step 5: Test the overall system performance using the calculated robust controller $K(s)$ under different kinds of disturbances
 where;

- γ_{opt} is optimum H_∞ -norm value :
- Itisys stores the state-space realization of system as the system matrix
- hinflmi computes the H-infinity performance when the system is controlled by $K(s)$
- slft forms the linear fractional interconnection of the two systems
- K is an optimal output-feedback controller

5. GENATIC ALGORITHM, ROBUST PID/ H_∞ CONTROLLERS

Genetic Algorithm (GA) is a stochastic search algorithm similar to the mechanism of natural selection. GA is used mainly to approximate the global optimum of an objective function (cost function or performance index), called *fitness*, that may contain several optimum points, and where a set of parameters, called *population*, that optimizes the objective function (*fitness*) has to be determined. Each member of the population, called *chromosome*, takes the form of a *binary string* of binary bits. The *chromosome* is then tested to find its fitness through its substitution into the fitness function that represents the environment in the biology counterpart. Moreover, it searches for many optimum points in a parallel fashion.

GA requires first a definition of a search interval and a selection of an initial population, randomly chosen inside the search interval, then finally, an iterative application of the three main steps; reproduction, crossover, and mutation, until convergence (stabilization of the fitness function) is obtained.

A. Robust PID via GA

Simple linear controllers are normally preferred over complex linear controllers for linear time-invariant plants. For this reason there is a desire to have a method available for designing a low-order controller for high-order plants obtained from RH_∞ control theories. A choice of a relatively low-order structure and popular controller which is ideally practical for industry such as PID is strategic. Hence, the objective of the proposed design is to tune the parameters

of a PID controller to achieve the same robustness as the standard RH_∞ output feedback control design. The resulting controllers are PID/ H_∞ with a reduced order.

B. Transfer Function [28]:

$$K_{PID}(s) = K_p + \frac{K_I}{s} + K_D s \quad (13)$$

Where K_p , K_I and K_D represent the gain parameters of the controller.

The state equations of the controlled power system by PID can be written in the vector-matrix differential equation form as

$$\dot{\mathbf{x}}_{clPID} = \mathbf{A}_{clPID} \mathbf{x} + \mathbf{B}_{clPID} \mathbf{u} \quad (14)$$

The controlled system matrices with PID controller are given in Appendix A.

C. Optimization Formulation

The optimization problem is thus defined to find K_p , K_I and K_D that minimizes the cost function through the GA optimization technique:

D. Objective Function for RH_∞ Controller

The cost function to be minimized is represented by the H_∞ -norm of the transfer matrix from w to z , i.e.,

$$J = \left\| G_{zwH_\infty} - G_{ZWPID} \right\|_\infty^2 \quad (15)$$

The H_∞ -norm of a stable transfer function $G(s)$ is its largest input/output Random Mean Square (RMS) gain over all nonzero input u values,

$$\|G\|_\infty = \sup_{\substack{u \in L \\ u \neq 0}} \frac{\|z\|_L}{\|w\|_L} \quad (16)$$

where L is the space of signals with finite energy, z measured output and w the disturbance. Basically, this is a disturbance rejection problem. In other words, it is a problem of minimizing the effect of the worst-case disturbance on the output. It is also defined as the maximum of the system largest singular value over all frequencies.

6. SIMULATION RESULTS

The digital simulation results are obtained using MATLAB Platform. The proposed system is tested under two cases one for normal loading and normal system parameters and another case using wide range parameters and change in demand loading as power system is always changed .

A. System with Centralized H_{∞}

Where all the units are connected to the same controller and an integrator is added in system feedback to reach zero steady state error. It is found that the controller size have similar size to the system size which is not practical at all.

B. System with DRH H_{∞}

This controller design technique is characterized by designing the controller of each unit alone (Diesel – Wind – Hydro), then combine it together and see the system output and each unit power output. An integrator must be added parallel to each controller to ensure system zero steady state error. It is found that the each unit controller have similar size to the each unit size which is difficult to be done in some practical condirions.

C. System with PID/ H_{∞}

Genetic Algorithm (GA) is used to minimize J in (15) and (16) to get the optimum values of the PID/ H_{∞} controllers in each unit Diesel-Wind-Hydro respectively. In these optimizations the data used for GA are:

1- Lower Limit: [1 1 0.01 1 1 0.01 0.1 0.1 0.01]

2-Upper Limit: [1000 1000 100 1000 1000 100 1000 1000 100]

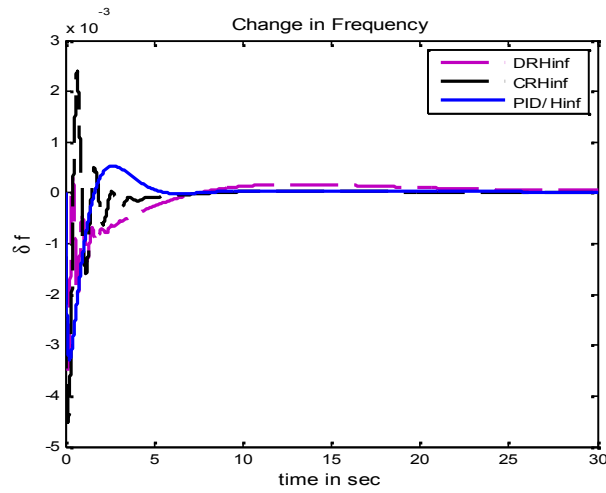
The optimum parameters values of these controllers calculate using one initial population is given in Table 1.

TABLE 1:Parameter Values of PID/ H_{∞}

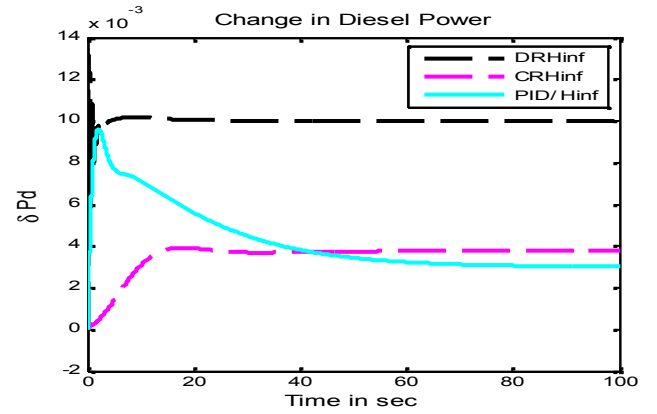
Parameter s	Diesel	Wind	Hydro
K_P	25	228	0.1
K_I	10	13	0.1
K_D	5	0.2	0.05

Case1: Comparison between Proposed Controllers at Normal loading

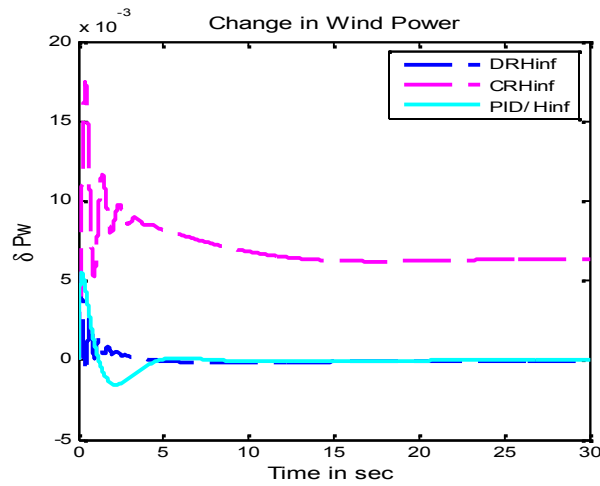
Comparisons between the dynamic responses of the system controlled by CRH_{∞} , DRH_{∞} and PID/ H_{∞} are shown in Fig 2(a,b,c,d) when the system is subjected to a 0.01 pu increase in demand power ΔP_L . The dynamic responses illustrated by Fig. 2(a) show the effectiveness of PID/ H_{∞} more than DRH_{∞} and CRH_{∞} with smaller overshoot and small settling time. H_{∞} is used mainly to decrease the effect of the disturbance on the system. In CRH_{∞} it is used to decrease the effect of the disturbance from the whole system while in DRH_{∞} it is used to decrease the effect of disturbance from each unit. CRH_{∞} has the highest undershoot -0.0045.



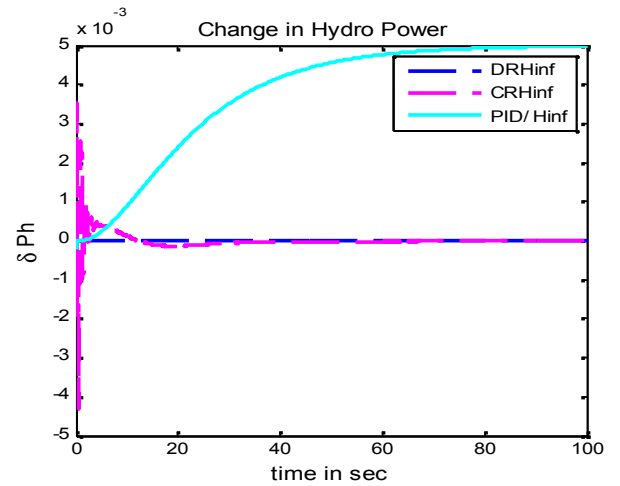
(a)



(b)



(c)

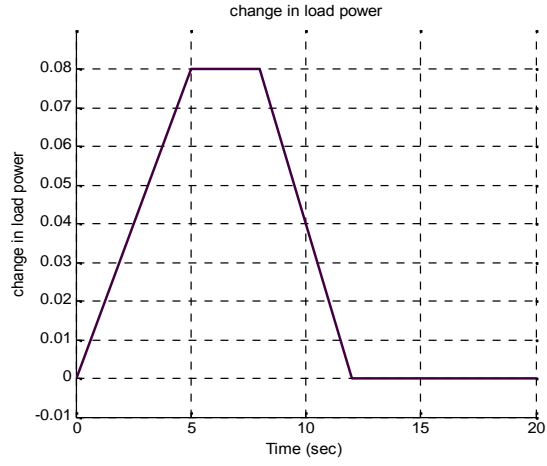


(d)

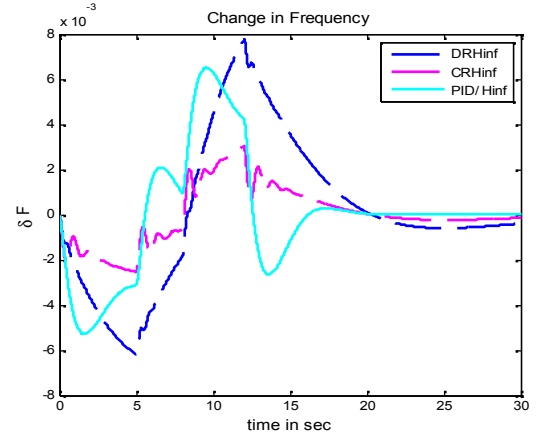
Fig. 2. Step-response for $P_d=1\%$ with 100% increase in system parameters

Case2: Wide Parameter Variation

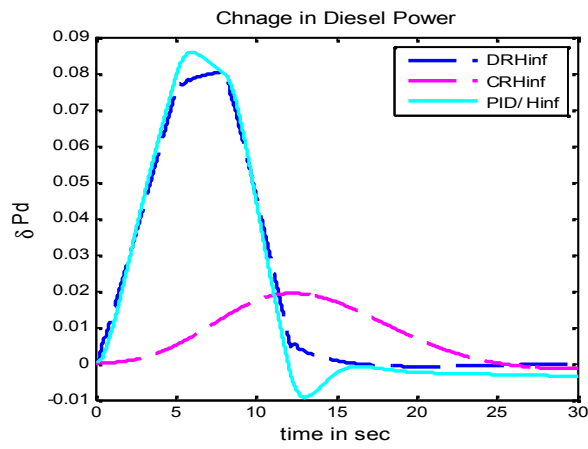
In this case 20% increase in system parameters (T_w , F , R_2 , T_{D1} , T_{D2} , K_D , T_{D3} , K_{p2} , T_{p2} , K_{p3} , T_{d3} , K_{Pc} , T_{D4} , K_{PL} , T_P). In the same time the compensated system is subjected to signal in Fig.3a, under this case the responses are found in Fig.3 (b,c,d,e). it is clearly seen that the controllers overcome these variations and give good results with a small settling time, thus indicating the effectiveness of these controllers over a wide range of parameter variation and change of operating conditions. The controller parameter values are still constant and are calculated using normal system parameters.



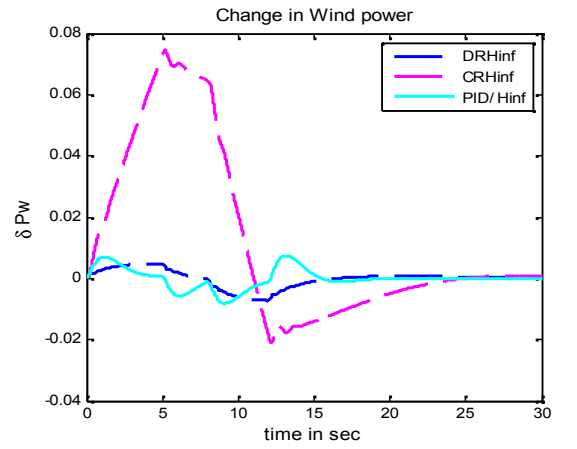
(a)



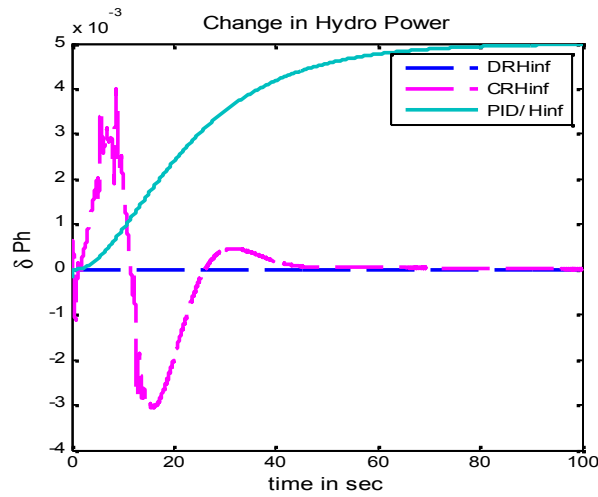
(b)



(c)



(d)



(e)

Fig. 3. Responses for P_d in (a) with 120% increase in system parameters

7. CONCLUSION

This paper has proposed and provided the design steps of three robust output-feedback controllers. The first and second controllers are CRH_∞ and DRH_∞ controllers. The third controller is robust PID/ H_∞ which is useful in industry and simple structure applications. The latter are proposed to achieve the same robust performance as DRH_∞ controllers. The first and second controllers have been solved using LMI. The effects of the LMI tuning variables RH_∞ controllers on the system dynamic performance have been presented and discussed. RH_∞ control theories and GA optimization technique are developed to compute the optimal parameters of the PID/ H_∞ controller. The cost functions of the optimization problems are represented by RH_∞ norms.

From the simulation results, it is clear that the system equipped with the three proposed controllers allows a better performance for improving the transients against diverse disturbances and useful to holding closed-loop stability and formulation of physical control constraints damping characteristics and shows better response. The comparison between the three controllers can be shortly summarized as follows:

The RH_∞ controllers have:

- 1- a similar size as the plant that may be of higher order and thus creates difficulty in implementation in large systems.
- 2- tuning variables of LMI

The PID/ H_∞ controller have:

- 1- a lower size order, ideally practical for industry, easier of implementation and operating as a robust RH_∞ controllers
- 2- rapid tracking of the different disturbances and showing good performance

Finally, the results prove that the proposed CRH_∞ , DRH_∞ and PID/ H_∞ are very useful in designing controllers for hybrid power system.

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8. Appendix

A. System and Controller matrices

i) Centralized H

a) System matrices

$$\begin{aligned}
 \mathbf{A} &= \begin{bmatrix} -7.5394 & 0.3735 & 0 & 25.4032 & 0 & 0 & 0.33 & 0 & 0 & 0 & 0 \\ 7.4700 & -0.6225 & 0.1120 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 24.39 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5.0403 & 0 & 0 & 0 & 0 & -3.78 & 0.1 & 0 \\ -1 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -24.39 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.33 & 40. & 0 & 0 & 0 \\ -0.2 & 0 & 0 & 0 & 0.1 & 0 & 0 & -40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & -0.16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \\
 \mathbf{B} &= \begin{bmatrix} -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0037 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0.005 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \mathbf{B}_1 + \mathbf{B}_2 \\
 \mathbf{C} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 &= \mathbf{C}_1 + \mathbf{C}_2 \\
 \mathbf{D} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

b) H System Controller matrices

The Desired H^∞ -norm: $\gamma = 0.1$

Optimum H^∞ -norm: $\gamma_{\text{opt}} = 10.64$

**CL=-1212.8,-702.67,-40.002,-44.946-16.058i,-44.946+16.058i,-7.9119,-26.699-24.449i,-
26.699+24.449i,-7.6892-0.91053i,-7.6892+0.91053i,-6.573,-2.0072,-0.5104,-0.28539-
0.085781i,-0.28539+0.085781i, -0.62184, -0.33193, -0.04855, -0.15589, -1, -24.39, -1**

$$A_k = \begin{bmatrix} -432.85 & -161.21 & -5.5711 & -68.611 & -23.33 & -110.92 & 34.981 & 67.98 & 9.5529 & -118.39 & 3513.3 \\ -115.99 & -44.023 & 0.94923 & -18.387 & -6.5807 & -27.978 & 8.4024 & 22.276 & 1.9193 & -32.603 & 942.36 \\ 4.0435 & 0.97918 & -34.979 & -9.5518 & -0.81117 & 4.5205 & 7.9023 & -9.9721 & 21.45 & 3.139 & -29.126 \\ -21.284 & -11.852 & 0.79686 & -24.833 & -0.43192 & -44.736 & 17.852 & -75.814 & -3.8698 & 12.345 & 211.1 \\ 1.7089 & -0.26612 & 3.87 & -2.9354 & -1.0842 & -1.5567 & -2.5203 & -1.0621 & -0.92449 & 1.382 & -8.7184 \\ -11.236 & -7.87 & -9.1508 & -27.541 & 3.9231 & -81.677 & 21.834 & -144.48 & -10.393 & 29.904 & 147.22 \\ 13.415 & -0.14996 & -7.9915 & -23.619 & -4.8071 & -18.957 & -27.16 & -17.082 & 0.83016 & 12.604 & -82.23 \\ 51.527 & 11.048 & -24.773 & -64.44 & 31.777 & -283.45 & 151.04 & -626.66 & -44.757 & 141.99 & -214.73 \\ 0.17751 & 3.7471 & -24.714 & 12.037 & 5.9346 & -13.416 & 10.338 & -31.041 & -20.612 & 6.1496 & -8.4333 \\ -13.863 & -8.1486 & 4.5687 & -23.301 & -9.8827 & 6.244 & -38.052 & 52.782 & 6.1128 & -19.497 & 121.57 \\ 79.481 & 29.875 & 4.6603 & 11.794 & 4.1842 & 17.372 & -6.7179 & -15.558 & -2.6588 & 21.548 & -763.32 \end{bmatrix}$$

$$B_k = \begin{bmatrix} 3.5602 & -0.19013 & -4.4835 \\ -6.3302 & 1.5078 & 9.5396 \\ -2.3081 & -34.283 & -5.4056 \\ -10.58 & -1.1667 & 46.07 \\ -4.2772 & 2.9256 & 10.215 \\ -23.066 & 2.0149 & -30.675 \\ -46.405 & -2.906 & 0.4459 \\ -3.6759 & 1.7226 & 17.094 \\ 19.925 & -17.512 & 25.372 \\ -40.306 & 2.5858 & 85.224 \\ -0.11682 & 4.065 & 40.81 \end{bmatrix}$$

$$C_k = \begin{bmatrix} -455.55 & -170.26 & -5.7433 & -74.503 & -24.264 & -122.71 & 39.445 & 59.834 & 9.0169 & -122.19 & 3706.4 \\ -121.5 & 210.61 & -1662.3 & 651.6 & 202.76 & -383.13 & -825.75 & -7.6983 & 1303.2 & 288.61 & -92.909 \\ -30.86 & -7.1676 & 14.732 & 32.434 & -17.973 & 152.1 & -82.989 & 340.37 & 24.879 & -79.523 & 142.1 \end{bmatrix}$$

$$D_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

ii) Decentralized H

(I) Diesel unit

a) Unit matrices

$$A = \begin{bmatrix} -0.069444 & 0 & 0 & 0.33333 & 0 \\ 5 & 0 & 0 & 0 & 0 \\ -1 & -1 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & -0.33333 & 40 \\ -0.2 & -0.2 & 0.1 & 0 & -40 \end{bmatrix}$$

$$B = B_1 + B_2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & -0.2 \end{bmatrix}$$

$$C = C_1 + C_2 = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

b) H unit controller matrices

desired=0.1

optimal=10.362

$$A_k = \begin{bmatrix} -0.45136 & -0.78864 & 5.0002 & 2.2897 & 18.634 \\ 0.61726 & 0.47731 & -6.2813 & -3.8905 & -22.512 \\ 0.82263 & 1.7813 & -8.8732 & -5.6017 & -25.39 \\ -0.55843 & -4.4923 & 1.1598 & -16.179 & 5.9781 \\ -3.5855 & 0.19238 & 17.189 & 108.53 & -243.65 \end{bmatrix}$$

$$B_k = \begin{bmatrix} -0.84537 \\ -0.17383 \\ -0.86806 \\ 4.9504 \\ 13.644 \end{bmatrix}$$

$$C_k = [-0.48434 \quad -2.8536 \quad 17.544 \quad 8.413 \quad 67.199], D_k = [0]$$

CL= -244.2, -39.956, -17.893, -4.895, -0.63409 -0.44711i, -0.63409 +0.44711i, -0.04653, -0.29312, -0.45008, -0.5402

(2) Wind unit

a) Unit matrices

$$A = \begin{bmatrix} -7.5394 & 0.3735 & 0 & 0 & 0 & 0 \\ 7.47 & -0.6225 & 0.1344 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 24.39 & 0 \\ 7.47 & -0.3735 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.00375 & -24.39 & 0.5 \\ 0 & 0 & 0 & 0.005 & 0 & -1 \end{bmatrix}$$

$$B = B_1 + B_2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.00375 \\ 0 & 0.005 \end{bmatrix}$$

$$C = C_1 + C_2 = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ -0.3735 & 7.47 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

b) H unit Controller matrices

desired=0.1

optimal=73.19

$$A_k = \begin{bmatrix} -2.3699 & -0.044948 & 0.15267 & 2.1154 & 1.609 & -0.60255 \\ -0.35487 & -6.8417 & 4.9235 & -0.14795 & 4.6589 & -48.744 \\ -0.12956 & -1.1474 & -1.2782 & 0.13167 & -0.11289 & -0.46828 \\ 5.3485 & 0.10817 & -0.25739 & -4.6712 & -2.7322 & 2.2184 \\ -38.456 & 1.2708 & -0.42685 & 30.419 & -1.776 & 0.68387 \\ -4.7909 & 5.9181 & 1.831 & 3.5105 & 2.3041 & -16.472 \end{bmatrix}$$

$$B_k = \begin{bmatrix} 0.35945 \\ -0.0092077 \\ 0.014015 \\ -0.88479 \\ 6.3812 \\ 0.82071 \end{bmatrix}$$

$$C_k = [69.591 \quad 1306 \quad -957 \quad -1.9589 \quad -938.75 \quad 9924.2], D_k = [0]$$

$CL = -24.386, -11.735 - 16.24i, -11.735 + 16.24i, -4.3267 - 11.346i, -0.63409, -4.3267 + 11.346i, -7.9235, -1.2737, -1.001 - 0.010984i, -1.001 + 0.010984i, -0.25078, -0.00029288, -0.00084101.$

(3) Hydro unit

a) Unit matrices

$$A = \begin{bmatrix} -0.069444 & -2 & 6 & -6.25 & 0 \\ 0 & 0.4 & -1.2 & 1.25 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & -2 & 6 & -6.5 & -1.25 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = B_1 + B_2 = \begin{bmatrix} -1 & -2 \\ 0 & 0.4 \\ 0 & 1 \\ 0 & -2 \\ 0 & 0 \end{bmatrix}$$

$$C = C_1 + C_2 = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0.4 & -1.2 & 1.25 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

b) H unit controller matrices

$desired=0.1$

$optimal=72$

$$A_k = \begin{bmatrix} -2.1306 & -2.2821 & 0.075314 & -10.316 & -21.086 \\ 0.4962 & -4.322 & -2.7071 & -13.694 & -26.159 \\ 1.2596 & -11.838 & -10.045 & -36.76 & -71.053 \\ 2.0581 & -10.682 & -9.39 & -33.138 & -63.276 \\ -160.88 & 81.656 & 17.46 & 4.6887 & -23.503 \end{bmatrix}$$

$$B_k = \begin{bmatrix} -0.35816 \\ 1.8044 \\ 5.3182 \\ 5.1136 \\ -110.49 \end{bmatrix}$$

$$C_k = [0.72082 \quad 4.3125 \quad 1.8134 \quad 17.294 \quad 35.225] , D_k = [0]$$

$$CL = \begin{bmatrix} -41.521, -25.511, -9.865, -2.8946, -0.12893 -1.1085i, -0.12893 + 1.1085i, -0.53635 - 0.19137i, -0.53635 + 0.19137i, -8.1401e-007, 0.1857 \end{bmatrix}$$

B. System parameters

Table 1: System Parameters Definition

Parameter	Defination
T_w	Water starting time
F	Temporary droop
R_2	Permanent droop
T_{D1}	Time between switching valve and produce torque
T_{D2}	low pressure reheat time
K_D	High pressure stage rating
T_{D3}	Generator delay time
K_{p2}	Hydraulic pitch actuator Gain
T_{p2}	Hydraulic pitch actuator time delay
K_{p3}	Data fit pitch actuator gain
T_{d3}	Data fit pitch actuator time delay
K_{Pc}	Fluid coupling gain
T_{D4}	Turbine time delay
T_p	Power system time constant
K_{PL}	Power system gain

Table 2 System Data

Stimulation parameters: $K_D=0.4$, $T_{D1}=1$, $T_{D2}=2$, $T_{D3}=0.025$, $T_{D4}=3$, $k_{Pc}=0.08$, $K_{p3}=1.4$, $K_{p2}=1$, $K_{p1}=1.25$, $T_{p1}=0.6$, $T_{p2}=0.041$, $T_{d3}=1$, $K_p=120$, $K_d=4$, $K_i=5$, $f=50$, $T_w=1$, $R_2=2.4$, $K_{PL}=72$ and $T_p=14.4$.