**Military Technical College**
**Kobry El-Kobbah,**
**Cairo, Egypt**

**9th International Conference**
**on Electrical Engineering**
**ICEENG 2014**

# Performance of OFDM-System in the Presence of Jamming and Carrier Frequency Errors

*By*

Karim A. Mansour* Ehab M. Shaheen * M. Soliman* Ahmed A. Elmahdy**

***Abstract:***

In this paper an investigation to the performance of single input single output (SISO) – orthogonal frequency division multiplexing (OFDM) system in both additive white Gaussian noise (AWGN) and multipath fading channels in the presence of jamming is provided. The effects of two types of jamming techniques; Partial band and broadband jamming on the performance of OFDM system are presented and investigated. The impact of symbol time offset on the performance of OFDM system is also investigated in the considered channels. A closed form analytical expression for the symbol error rate for different values of the carrier frequency offset is derived.

***Keywords:***

OFDM, Partial band jamming, broadband jamming, carrier frequency errors, multipath fading channels.

---

* Egyptian Armed Forces
** German University in Cairo

## ***1. Introduction:***

In the last few years OFDM has been object of increasing interest, in relationship to different applications, since it assures high data rate transmissions immune to channel dispersion[1]. It is well know that if the channel impulse response is much longer than the symbol duration, the received signal will be distorted in time. Nevertheless, for modern multimedia applications operating with very high data rate communications the required signal bandwidth can result much greater than the channel coherence bandwidth so that distortion effects are severe[1].

To contain such distortion it is necessary to use equalization systems, whose structure results more and more complex as the ratio among the channel delay spread and the symbol period increases.

The OFDM modulation scheme offers an alternative solution to deal this problem. High data rate transmission over mobile or wireless channels is required by many applications. However, the symbol duration reduces with the increase of the data rate, and dispersive fading of the wireless channels will cause more intersymbol interference (ISI) if single-carrier modulation, such as in time-division multiple access (TDMA) or Global System for Mobile Communications (GSM)[2].

The performance of OFDM systems may be severely deteriorated due to the presence of strong jamming signals. In [3] a power efficient asynchronous off-tone jamming attacks on OFDM is presented, where they show that off-tone jamming attacks have severe impact on OFDM systems. In [4] an analysis for the performance of carrier interferometry/orthogonal frequency division multiplexing (CI/OFDM) in the presence of multipath fading as well as jamming is presented. Yet, in [5] an analysis of the joint effect of nonlinear distortion which is produced by nonlinear high power amplifiers (HPAs) and jammer on the performance of a Multiple Input Multiple Output (MIMO)- OFDM system in Rayleigh fading channels is presented

In order to contribute in such research this paper investigates the impact of two types of jamming techniques partial band jamming (PBJ) and broadband jamming (BBJ) along with the effect of synchronization errors on the bit error rate (BER) performance of OFDM systems.

This paper is organized as follows. Section II depicts the used system model, including the jamming signal model and the channel. An analysis of the received signal is depicted in section III. Section IV depicts the BER performance analysis under the impact of jamming. Numerical and simulation results are presented in section V. Finally, section VI draws the conclusions.

## 2. System Model:

Let us consider a general OFDM signal transmitted by the interferer [6]

$$S_{ofdm}(t) = Re\left\{\sum_{i=-\infty}^{\infty}\sum_{n=1}^{N} A_n^i\, g(t - iT_s) e^{j2\pi f_n(t-\tau)+\psi}\right\} \quad (1)$$

Where N is the number of subcarriers, $A_n^i$ symbols for the nth subcarrier, $T_s$ is the symbol duration, g(t) is the transmitted pulse waveform and $f_n = f_0 + \Delta f\left(n - 1 - \frac{N-1}{2}\right)$ are the subcarrier frequencies equally spaced by Δf and centered at $f_0$ . Since we assume that the interferer is asynchronous with respect to the desired signal, we model τ as a random time delay and ψ as a random variable (r.v.) uniformly distributed in [0, 2π).

The jamming signal, denoted by J(t), is a stationary zero mean complex Gaussian process active on a fraction ρ of the signal bandwidth. The assumption of Gaussian jamming signal is considered in several articles as [7] and [8].We assume that the jammer has a flat power spectral density (psd) $2Q_o$. When the jammer is present, it covers a fraction ρ of the signal bandwidth $W_s$. In broad-band jamming, the jammer should occupy the whole communicator bandwidth. Thus, broad band jamming is considered as a special case of the partial band jamming by setting the fraction ρ to be equal one.

The channel impulse response for OFDM signal can be written as

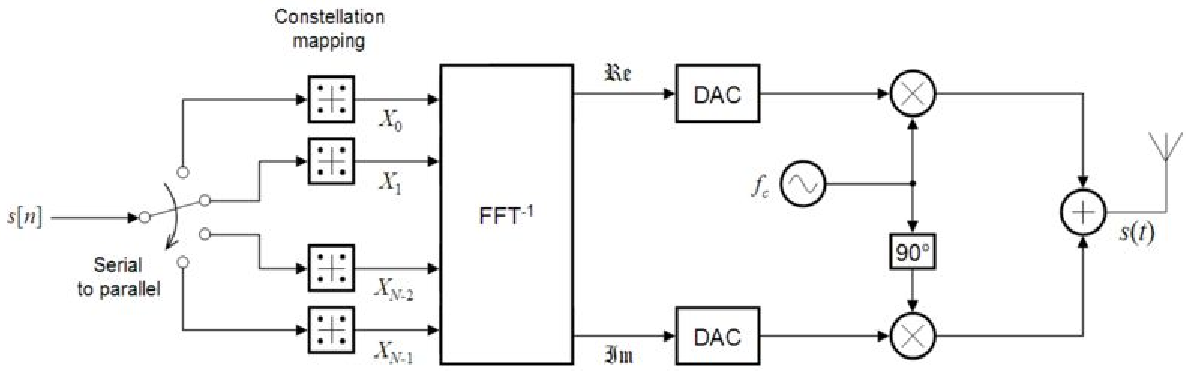
$$\mathbf{h_s(t) = \alpha_s \delta(t - \tau_s)} \quad (2)$$

where $\alpha_s$ is the Rayleigh distributed channel gain and $\tau_s$ is the corresponding time delay. The received signal can be written as

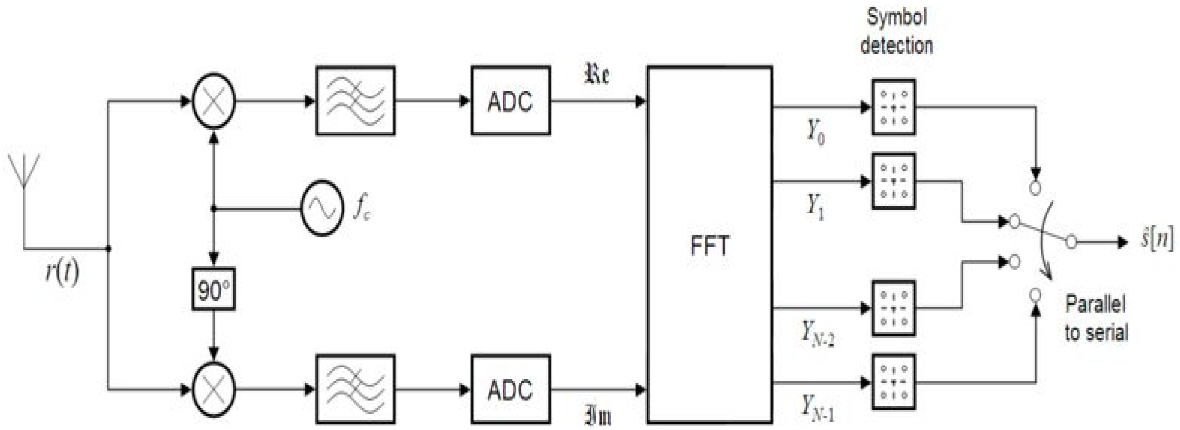
$$\mathbf{r(t) = S_r(t) + J(t) + n(t)} \quad (3)$$

where n(t) is the additive white Gaussian noise (AWGN) with two sided power spectral density $N_o/2$.

Figure (1) depicts the traditional OFDM system model.




**a) OFDM Transimeter**




**b) OFDM Receiver**

**Figure (1) OFDM System Model.**

## *3. Performance Analysis:*

We begin with a derivation of the probability of error assuming OFDM transmission over an AWGN channel with jamming over subcarriers (m, m + 1, . . . , m +M − 1). In OFDM systems, the decision variable for the ith symbol (bit) is easily shown to correspond to

$$r_i = \begin{cases} As^{(i)} + J_i + n_i, & i \in \{m, m+1, \ldots, m+M-1\} \\ As^{(i)} + n_i & otherwise \end{cases} \quad (4)$$

where $A = \sqrt{E_b}$. The probability of error for OFDM in the presence of jamming in an AWGN channel therefore corresponds to

$$P(e) = \frac{N-M}{N}.Q\left(\sqrt{\frac{2E_b}{N_o}}\right) + \frac{M}{N}.Q\left(\sqrt{\frac{2E_b}{N_o + 2\sigma_j^2}}\right) \qquad (5)$$

In (8), the first term represents the contribution from carriers that do not experience jamming and the second term represents the contribution of "jammed carriers." It is evident that the average BER is dominated by the latter term. It is important to note that when the power of the jammed signal is very high, (8) converges to

$$P(e) = \frac{M}{N}.Q\left(\sqrt{\frac{2E_b}{N_o + 2\sigma_j^2}}\right) = \frac{M}{2N} \qquad (6)$$

In a flat fading channel, all subcarriers experience an identical fade. Thus, in OFDM, the decision variable for the ith symbol corresponds to

$$r_i = \begin{cases} A\,(s^{(i)} + J_i + n_i, & i \in \{\mathbf{m}, \mathbf{m}+\mathbf{1}, \dots, \mathbf{m}+\mathbf{M}-\mathbf{1}\} \\ A\,(s^{(i)} + n_i & otherwise \end{cases}$$

(7)

It is well known that the corresponding probability of error for the ith symbol (assuming α is Rayleigh distributed) is

$$P_i(e) = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_i/_2}{1 + \bar{\gamma}_i/_2}}\right) \qquad (8)$$

where $\bar{\gamma}_i$ is the average SNR for the ith symbol, defined as

$$\bar{\gamma}_i = \frac{E_b}{\sigma_i^2} E\{\alpha^2\} \qquad (9)$$

where $\sigma_i^2$ refers to $\sigma_i^2 = \sigma_j^2 + \frac{N_o}{2}$

if $\mathbf{i} \in \{\mathbf{m}, \mathbf{m}+\mathbf{1}, \ldots, \mathbf{m}+\mathbf{M}-\mathbf{1}\}$. Hence, the average BER of OFDM in a flat fading channel corresponds to:

$$P(e) = \frac{N-M}{N} \cdot \frac{1}{2}\left(1 - \sqrt{\frac{\frac{E_b}{N_o}E\{\alpha^2\}}{1+\frac{E_b}{N_o}E\{\alpha^2\}}}\right) + \frac{M}{N} \cdot \frac{1}{2}\left(1 - \sqrt{\frac{\frac{E_b}{N_o+2\sigma_J^2}E\{\alpha^2\}}{1+\frac{E_b}{N_o+2\sigma_J^2}E\{\alpha^2\}}}\right) \quad (10)$$

In ( ), the first term represents the contribution of carriers without jamming and the second term represents the contribution from “jammed carriers”.

## ***4. Synchronization error Analysis:***

The sampled signal for the *l*th subcarrier after the receiver fast Fourier transform processing can be written as:

$$Y_l = X_l H_l I(0) + \sum_{k=-\frac{N}{2}, k\neq l}^{\frac{N}{2}-1} X_k H_k I(k-l) + N_l \quad (11)$$

where $X_l$ represents the transmitted complex QAM modulated symbol on subcarrier l and $N_l$ is a complex Gaussian noise sample. The coefficient $H_k$ denotes the frequency domain channel transfer function on subcarrier k, which is Fourier transform of the channel impulse response (CIR) h(τ) with maximal L taps. The coefficient $I(k-l)$ represents the impact of the received signal at subcarrier k on the received signal at subcarrier l due to the residual carrier frequency offset [9]:

$$I(k-l) = e^{j\pi((k-l)+\Delta f)(1-1/N)} \frac{\sin(\pi((k-l)+\Delta f))}{N\sin\left(\frac{\pi((k-l)+\Delta f)}{N}\right)} \quad (12)$$

where Δf is the residual carrier frequency offset normalized to the subcarrier spacing.

The preamble-based Least Square (LS) frequency domain channel estimation to obtain the channel state information on subcarrier l can be considered as:

$$\hat{H}_l = \frac{Y_{pream,l}}{X_{pream,l}} = H_l I(0) + \frac{\sum_{k\neq l} X_{pream,k} H_k I(k-l) + N_l}{X_{pream,l}} \quad (13)$$

where $X_{pream,l}$, $Y_{pream,l}$ denote the transmitted and received preamble on subcarrier l. It is worth to be noted that the Gaussian noise of preamble part $\acute{N}_l$ has the same variance as $N_l$ of the data part.

In [10] the symbol error rate (SER) of a given quadrature amplitude modulation (QAM) can be written as

P↓e (x↓m ) = 1 − {[((g↓i − b↓(m, i)) 〚tan〛↑(−1) ((g↓r − b↓(m, r))/√(a↓m↑2 + (g↓i − b↓(m, i) )↑2 )))/
(14)

where for each constellation point $x_m$, the parameters $g_{r,min}$, $g_{r,max}$, $g_{i,min}$ and $g_{i,max}$ represent the lower and upper boundaries of the mth symbols Cartesian decision region.

Now the symbol error rate of OFDM systems with carrier frequency offset and channel estimation error in Rayleigh flat fading channels ($H_l = H_k = H$, $\forall$ k, l $\in$ [1, ...,N]) based on Eq. (14) will be derived.
Firstly, we can rewrite the channel estimates of subcarrier l as

$$\widetilde{H}_l = \frac{Y_{pream,l}}{X_{pream,l}} = I(0)H\left(1 + \frac{\Sigma_{k\neq l} X_{pream,k} I\,(k-l)}{I\left(0\right) X_{pream,l}}\right) + \frac{\acute{N}_l}{X_{pream,l}} \qquad (15)$$

The single-carrier transmission model without carrier frequency offset for Rayleigh flat fading channels can be written as

$$Y = hx + n \qquad (16)$$

where y, h, x and w denote the complex baseband representation of the received signal, the channel coefficient, the transmitted data and the Gaussian noise with variance $\sigma^2_n$ respectively.
In [6], the channel estimate $\hat{h}$ is assumed to be biased and used for zero forcing equalization as follows

$$z = \frac{y}{\hat{h}} \qquad (17)$$

with

$$\hat{h} = \alpha h + \delta \qquad (18)$$

where α denotes the bias in the channel estimates and $\delta$ is a zero-mean complex Gaussian noise with variance $\sigma_\delta^2$.

Then

$$\widehat{H}_l = \acute{\alpha}_l \acute{H} + \frac{\acute{N}_l}{X_{pream,l}} = \acute{\alpha}_l \acute{H} + \acute{\delta}_l \tag{19}$$

where

$$\acute{H} = I(0)H\,, \qquad \acute{\alpha}_l = 1 + \frac{\sum_{k\neq l} X_{pream,k} I(k-l)}{I(0) X_{pream,l}} \tag{20}$$

$\acute{\alpha}_l$ is a deterministic quantity with given subcarrier index l, a set of preamble symbols $X_{pream,k}$ and a fixed frequency offset.

Applying the same method above for the OFDM equation represented in equation (1) then:

$$Y_l = \acute{H}\left(X_l + \frac{\sum_{k\neq l} X_k I(k-l)}{I(0)}\right) + N_l \tag{21}$$

$$Y_l = \acute{H}\,\acute{X}_l + N_l$$

where we define the effective symbol $\acute{X}_l$, that is no longer a deterministic value but a stochastic quantity due to i.i.d. data symbols on subcarriers k $\neq$ l.

Then the effective symbol $\acute{X}_l$ can be decomposed as:

$$\acute{X}_l = X_l + \frac{\sum_{k\neq l} X_k I(k-l)}{I(0)} \tag{22}$$

where the second part of the equation represents the stochastic nature of $\acute{X}_l$ due to the random inter-carrier interference (ICI) part.

The inter-carrier interference term is assumed to be a complex zero-mean Gaussian random variable $ICI_l = p+jq$.

The symbol error rate on subcarrier l for the mth constellation point can be expressed by the following double integral

$$\hat{P}_e(X_{m,l}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{P_e\left(X_{m,l} + \mathbf{p} + \mathbf{jq}\right)}{2\pi\sigma^2_{ICI_l}} e^{-\frac{p^2+q^2}{2\sigma^2_{ICI_l}}} dp\,dq \tag{23}$$

where

$$\sigma^2_{ICI_l} = \frac{1 - |I(0)|^2}{2|I(0)|^2} \tag{24}$$

Finally, to obtain the general symbol error rate we have to average Eq.() over all subcarriers with index l and M-QAM constellation points with index m as follows

$$P_{se} = \frac{1}{MN} \sum_{l=-\frac{N}{2}}^{\frac{N}{2}-1} \sum_{m=1}^{M} \hat{P}_e(X_{m,l}) \tag{25}$$

## ***5. Research Analytical and Simulation Results:***

In this section, an analytical evaluation of the IEEE802.11a OFDM based system performance is investigated and validated with the aid of simulation. The IEEE802.11a used parameters values are: center frequency = 5.22GHz, the number of carriers = 64.

Figure (2) represents the BER performance of the OFDM system in AWGN channel in the presence of jamming for different SJR values. It can be seen that the performance can be severely degraded in the presence of jamming, for example at BER = 10-3 a SNR degradation can reach up to 1 dB at SJR = 10dB. Yet, for SJR = 0dB the OFDM system is completely saturated by the impact of jamming.

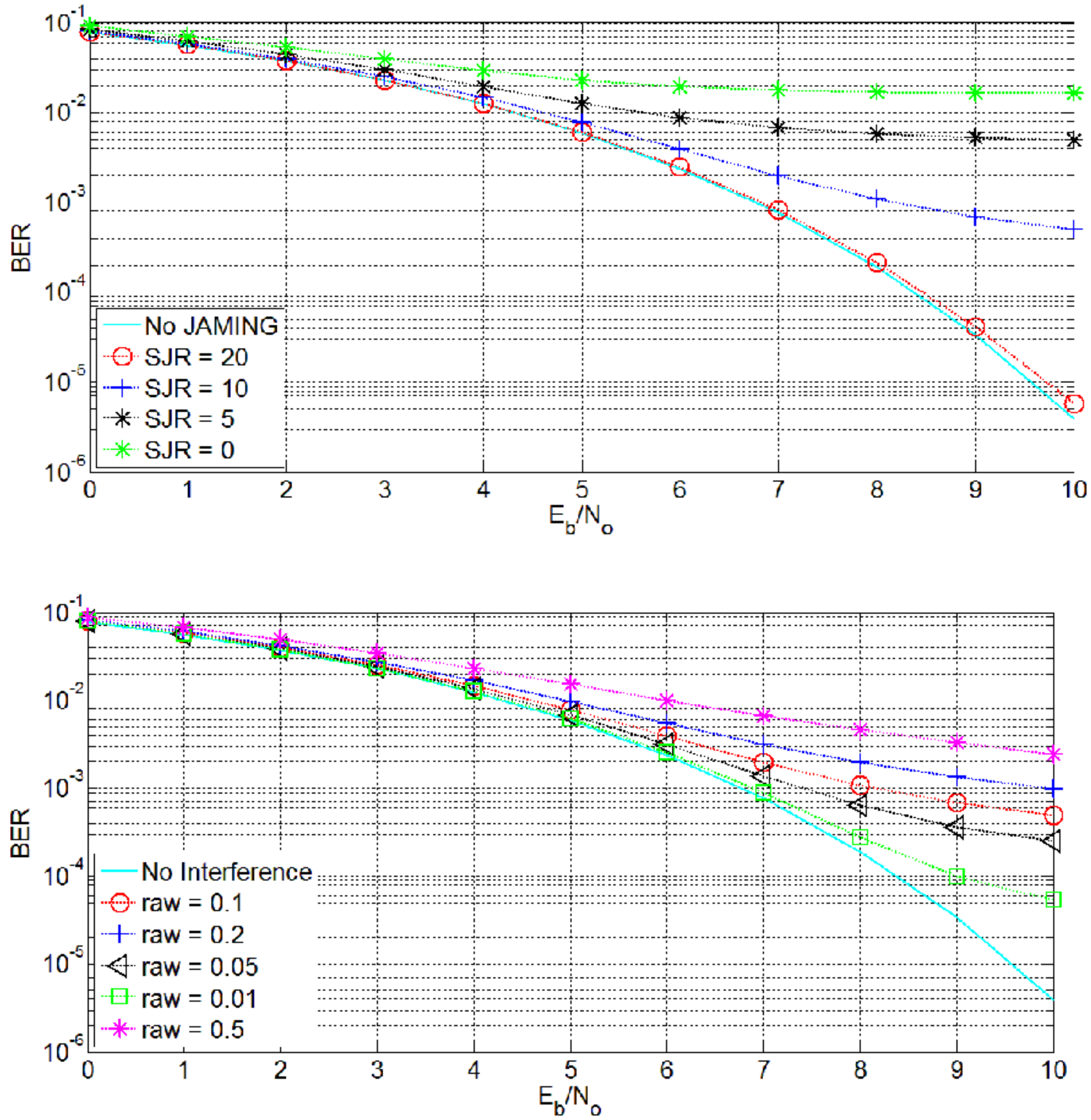



Figure (3) BER performance of the OFDM system in the presence of jamming in AWGN channel for different jamming techniques.

Figure (3) represents the BER performance of the OFDM system in AWGN channel in the presence of jamming for different jamming techniques (partial band and broad band jamming). It can be seen that the performance can be severely degraded in the presence of jamming for different partial bands , for example at BER = $10^{-3}$ a SNR degradation can reach up to 3 dB at raw = 0.2 . Yet, for raw = 0.5 the OFDM system is completely saturated by the impact of jamming.

Figure (2) BER performance of the OFDM system in the presence of jamming in AWGN channel

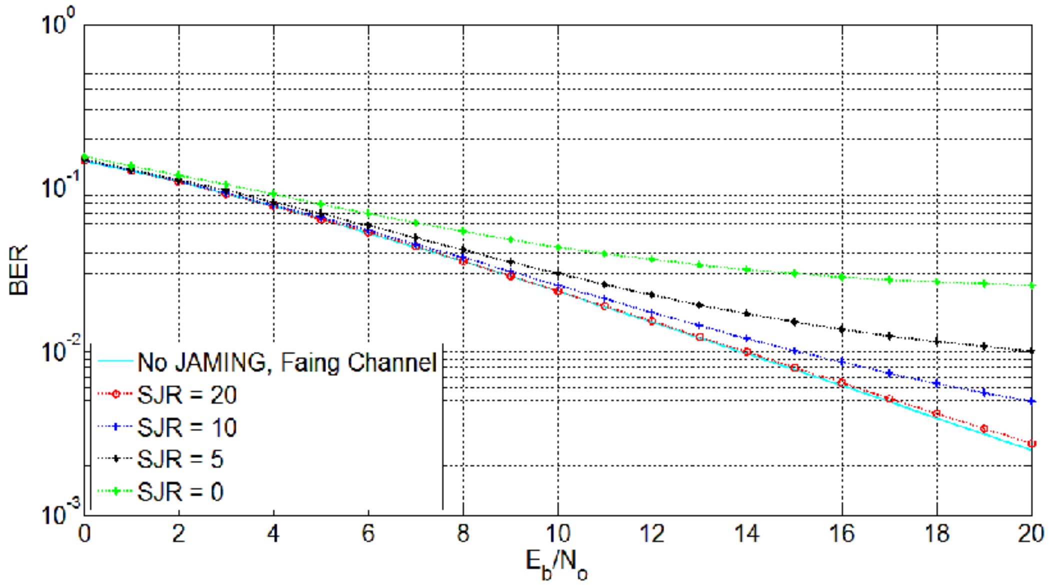



Figure (4) BER performance of the OFDM system in the presence of jamming in flat fading channel.

Figure (4) represents the BER performance of the OFDM system in fading channel in the presence of jamming for different SJR values. It can be seen that the performance can be severely degraded in the presence of jamming, for example at BER = $10^{-2}$ a SNR degradation can reach up to 6 dB at SJR = 10dB. Yet, for SJR = 0dB the OFDM system is completely saturated by the impact of jamming.

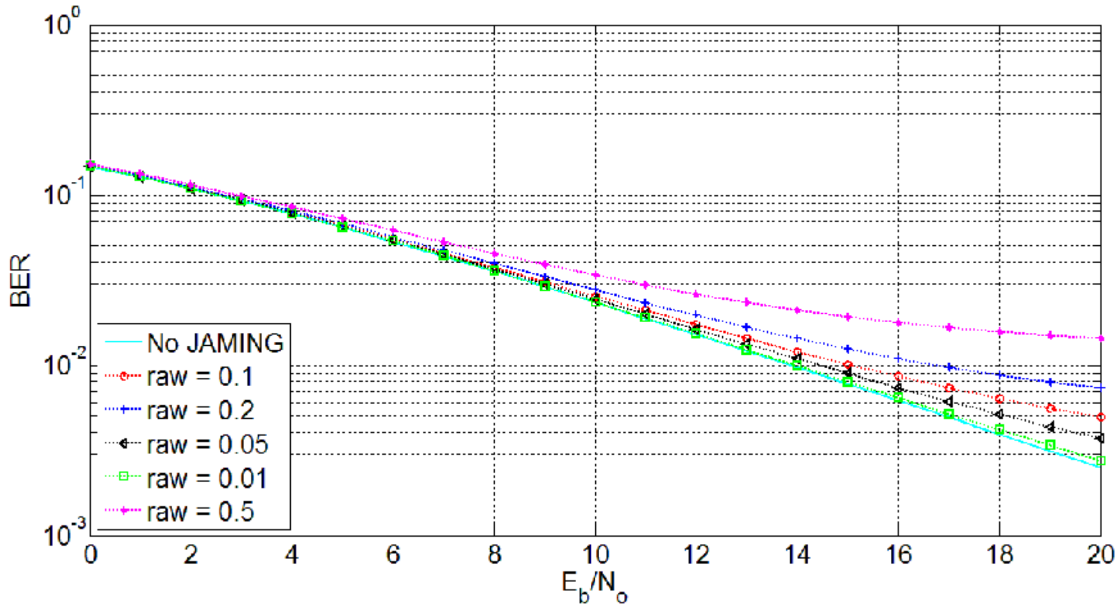



Figure (5) BER performance of the OFDM system in the presence of jamming in flat fading channel for different jamming techniques.

Figure (5) represents the BER performance of the OFDM system in fading channel in the presence of jamming for different partial bands. It can be seen that the performance can be severely degraded in the presence of jamming for different partial bands , for example at BER = $10^{-2}$ a SNR degradation can reach up to 6 dB at raw = 0.2 . Yet, for raw = 0.5 the OFDM system is completely saturated by the impact of jamming.

For the frequency synchronization analysis, The data is QPSK-modulated to different subcarriers, then transformed to a time domain signal by IFFT operation and prepended by a 16-tap long cyclic prefix. The data is randomly generated and one OFDM symbol preamble was used for channel estimation.

Figure (6) present the calculated symbol error rates vs. SNR with given carrier frequency offset Δf for Rayleigh flat fading channels.

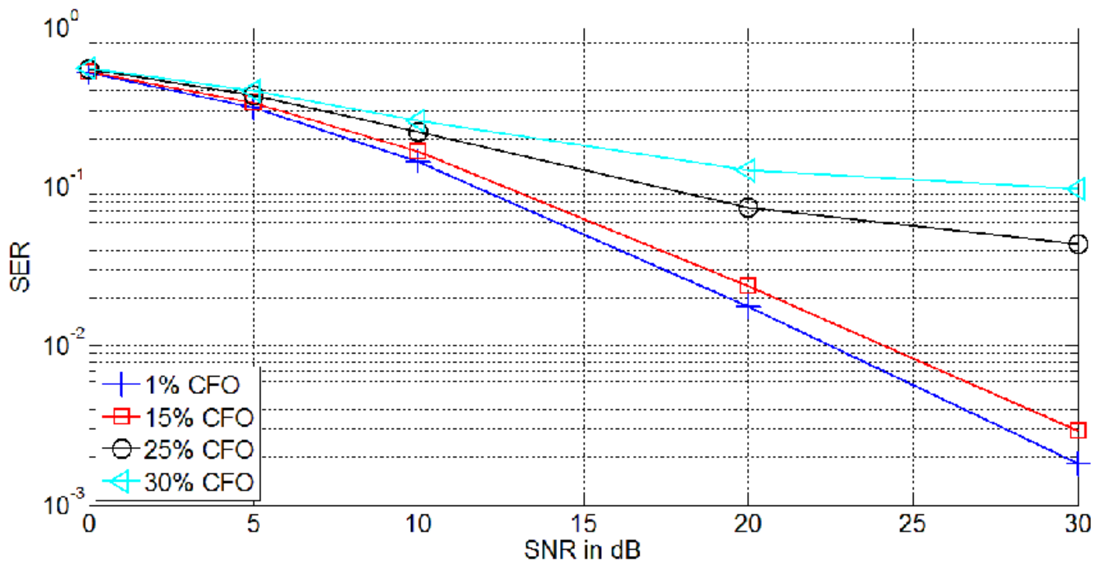



Figure (6) SER performance of the OFDM system in the presence of different CFO values in flat fading channel.

***c. Conclusion:***

The performance of the OFDM system has been investigated in the presence of two different types of jamming techniques which are the partial band and the broad band jamming in both AWGN and flat fading channels. The performance of the system has been analytically evaluated in the presence of jamming. It has been shown that the performance of the system is severely deteriorated due to the presence jamming.

Also, the symbol error rate performance has been analytically investigated in the presence of CFO and channel estimation error.

***References:***

[1] T. D. FUSCO, Synchronization Techniques for OFDM Systems, PhD, UNI VER SI TA` DEGLI STUDI DI NAPOLI FEDER I C O II, 2005.

[2] Y. S. Cho, MIMO-OFDM WIireless Communications with MATLAB, 2010.

[3] L. L. Chris Snow, and Robert Schober, "Impact of Tone Interference on Multiband OFDM," presented at the The IEEE 2006 International Conference on Ultra-Wideband, 2006.

[4] M. Zhiqiang Wu, IEEE, and Carl R. Nassar, Senior Member, IEEE, "Narrowband Interference Rejection in OFDM viaCarrier Interferometry Spreading Codes," IEEE Transactions on Wireless Communications, vol. 4, 2005.

[5] D. W. C. a. P. Das, "Effects of Jammer and Nonlinear Amplifiers in MIMO-OFDM with Application to 802.11n WLAN," IEEE, 2008.

[6] A. Giorgetti and D. Dardari, "The impact of OFDM interference on TH-PPM/BPAM transmission systems," IEEE 61st Vehicular Technology Conference, vol. 2, pp. 1037-1042, June 2005.

[7] Cheng Wang, Maode Ma, Rendong Ying, and Yuhang Yang, "Narrowband Interference Mitigation in DS-UWB Systems," IEEE Signal Processing, vol. 17, no. 5, May 2010.

[8] T. Zhao, Z. Yang and Y. Zhao "Partial-code-aided scheme for narrowband interference suppression in direct-sequence spread spectrum systems," IET Commun., 2010, Vol. 4, Iss. 18, pp. 2240–2251.

[9] K. Sathananthan and C. Tellambura, "Probability of error calculation of OFDM with frequency offset," In Proc. IEEE Transactions on Communications, volume 49, pages 1884–1888, November 2001.

[10] S. K. Wilson and J. M. Cioffi., "Probability density functions for analyzing multi-amplitude constellations in Rayleigh and Ricean channels," In IEEE Transactions on Communications, volume 47, March 1999.