Military Technical College

Kobry El-Kobbah,

Cairo, Egypt

8th International Conference on Electrical Engineering

ICEENG 2012

INS Alignment Using Onboard Computer Software for Missile Trajectory

Dr/ Ibrahim Ismail Arafa, Dr/ Ahmed Mohsen, Eng/ A.Elboraee
Guidance Department, Military Technical Collage (M.T.C.), Cairo, Egypt

Abstract

In this paper, the onboard computer software program was developed to predict the full trajectory of the missile in order to improve the accuracy at impact. For system in which time is not critical, the alignment process can be performed prior to launch by utilizing navigation data from the launch platform and calibrating the missile's INS to this reference. However, for systems that require rapid reaction time, significant pre-launch delays for alignment are not tolerable. In addition, due to low quality of the INS and its life cycle is short; hence every aborted mission will bring the INS closer to its mean time between failures (MTBF). These problems can be minimized by in-flight alignment (IFA). While IFA may seem to be less accurate and more complicated than alignment in the rest, it turns out that the ability of the carrier to maneuver during the IFA phase is a blessing in disguise since it enables the excitation of latent modes and thus, enhances their observability of the whole INS. The effects that maneuvers have on the estimability of the INS states during IFA were investigated in the past. But until now, there are many questions such as: what is the degree of observability? What limits the estimability of the system? What are the minimal additional measurements needed to turn the system into a completely observable one? And what are the useful numbers of maneuvers in IFA? , were not completely addressed. These questions are answered through this paper. We introduce the error model, which appropriately describes the INS behavior during IFA. We then substantiate its adequacy for consideration as a piece-wise constant system (PWCS) and then the simpler stripped observability matrix (SOM) can be used in the system observability analysis.

Introduction

The INS navigation loop provides continuous and reliable navigation solutions to the guidance and flight control loop for autonomous flight. With additional air data and engine thrust data, the guidance loop computes the guidance demands to follow way-point scenarios. The flight control loop generates actuator signals for the control surfaces and thrust vector. The whole Guidance, Navigation and Control (GNC) algorithm was implemented within an embedded flight control computer. The real-time flight test results show that the vehicle can perform the autonomous flight reliably with high accuracy [1].

The Onboard computer software program was developed to predict the full trajectory of the missile in order to improve the accuracy at impact. The primary purposes of missile launch surveillance are: 1) to provide a timely report of each occurrence of a missile launch, 2) to estimate launch/trajectory parameters, and 3) to estimate present and future missile trajectories as a function of time during flight. After the missile position and velocity vectors are obtained, the prediction starts with the time integration of equations of motion using a six parameter-state vector, which is composed of the position and velocity vectors, [2].

The integration procedure continues until the criterion of the missile range control is satisfied through onboard computer software. The guidance loop generates the guidance commands from the vehicle states and the desired waypoint information. It computes required vehicle speed with respect to the air, height and bank angle. Then the flight control loop (or autopilot) generates actuator control signals to make the vehicle follow the guidance demands as well as to stabilize the vehicle. The control outputs are fed to the control surfaces, or aileron, elevator and rudder, and thrust vector [3, 4].

1) Program objective (mission)

The basic program objective is to fly a typical mission profile as shown in fig (1) with an on target accuracy of $\pm 200m$. The flight path profile in the active phase approximately 59 sec. The reminder of 3 minute flight is a ballistic trajectory. The commnd of separation will arrive after the shut-off time of the engine within 5 sec. The payload objects are deployed in reentry phase over a distance about $25\,km$ from the ground. To meet target accuracy, the navigation system must provide accurate position and velocity input to the overall Guidance Navigation and Control System (GNCS) for the active phase of flight [5]. At the end of active phase, the program has no other propellant capability so the GNCS has no more control over the delta velocity of the payload objects.

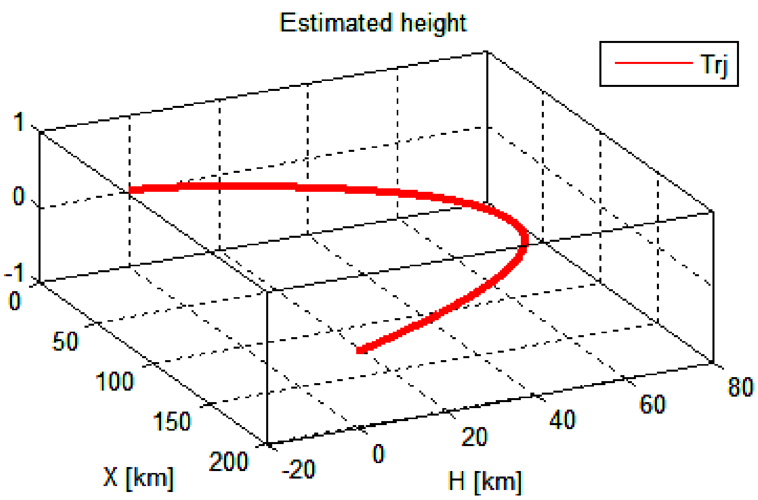


Fig (1) Mission profile

The navigation solution must therfore be highly accurate for the first 59 sec of flight (active phase) and thereafter, it provides useful information about the shape of the trajectory actually flown.

2) On-board Software

The On-Board Software (OBS) is a complex system with many functions. It performs the following functions as shown in fig (2): interface with ground support equipment, the majority of system test and checkout, ground processing, launch and all of the navigation, guidance, and control and sequencing functions required for flight. The OBS development was comprised of 3 major components: 1) the operating system, IMU digital signal processing code and ground alignment software and 2) ground checkout code developed uniquely for the program system and 3) the flight guidance, navigation, control and sequencing code. The flight navigation routines are dependent on inputs from the IMU sensors (gyros, accelerometers) [6]. The compensated sensor outputs, i.e. incremental angles and velocities, are accumulated and made available at 256 [Hz].

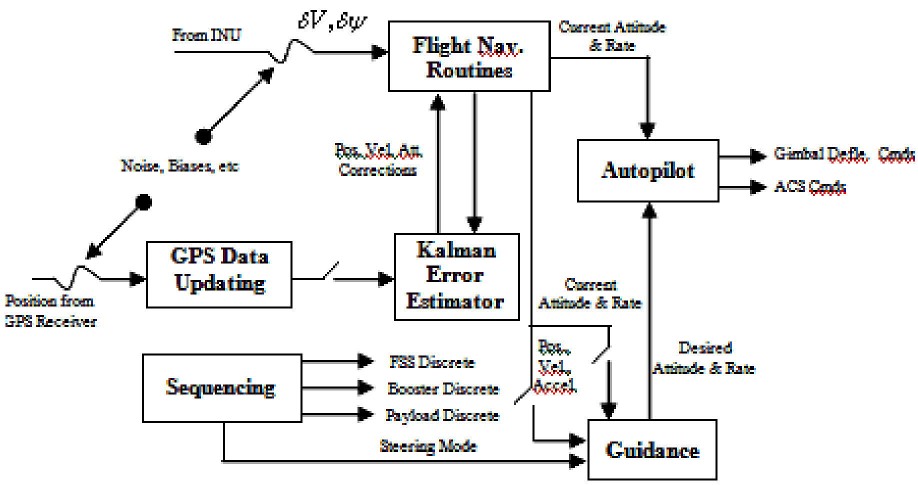


Fig (2) OBC flight functional diagram

Additional compensations are also implemented in the flight inertial navigation algorithm to compensate for the estimated instrument parameters via the System Error Estimator (SEE) updates [6,7]. After the final implementation integrates the guidance kit with the flight navigation solution via the Kalman filter**:** There are two modes of operation for the system: 1) ground test which assure the navigation system is healthy before launch and 2) navigation in flight.

In the ground test we compute the system azimuth, level orientation and auxiliary sensor compensation through the ground alignment time. This creates the initial attitude reference and gyro and accelerometer biases and scale factors which are used at the time of (Go Inertial) for the flight navigation system [7, 8].

3) Protocol of Mathematical tasks

Here we describe the block diagram of an operating program, functional modules and interrupt handlers as part of the Navigation software. The Navigation mathematical software (NMSW) performs the following functions:

- Support for INS initial alignment
- Solution of navigation problems: inertial velocity and coordinate reckoning.
- Interaction with navigation equipment and external users of navigation data.
- Support for inertial sensors calibration.

The interaction with users and external systems is provided using RS-232 interface. The NMSW is intended for operation as part of INS in the central processor.

a) Navigation Mathematical Software

Transformation matrix between Wander azimuth launch coordinate system and body-fixed system:

$$Mbt = \begin{pmatrix} \cos\vartheta\cos\psi & \sin\vartheta\cos\psi & \sin\psi \\ -\cos\vartheta\sin\gamma\sin\psi - \sin\vartheta\cos\gamma & -\sin\vartheta\sin\gamma\sin\psi + \cos\vartheta\cos\gamma & \sin\gamma\cos\psi \\ -\cos\vartheta\cos\gamma\sin\psi + \sin\vartheta\sin\gamma & -\sin\vartheta\cos\gamma\sin\psi - \cos\vartheta\sin\gamma & \cos\gamma\cos\psi \end{pmatrix}$$

The expressions for ϑ, EMBED Equati⌶n.3 , ψ orientation angles take the form:

$$\vartheta = \mathrm{arctg}\left(\frac{\mathrm{Mbt}_{12}}{\mathrm{Mbt}_{11}}\right);\ \gamma = \mathrm{arctg}\left(\frac{\mathrm{Mbt}_{23}}{\mathrm{Mbt}_{33}}\right);\ \psi = \mathrm{arctg}\left(\frac{\mathrm{Mbt}_{13}}{\sqrt{\mathrm{Mbs}_{11}^2 + \mathrm{Mbs}_{12}^2}}\right)$$

b) Earth Model

Ellipsoid of rotation with the axis coinciding with the Earth rotation axis can be used as the Earth navigational model relative to the systems, where the method of inertial navigation is implemented:

a is the semi major axis, b is the semi minor axis, f is the flattening of the ellipsoid·

$$e = \sqrt{[f(2-f)]};\ e^2 = \frac{a^2 - b^2}{a^2} \quad ; \quad f = \frac{a-b}{a};$$

WGS-84 international reference-ellipsoid with the following parameters is selected to be used in NMSW:

a = 6 378 137 m
f = 1/298.257223563
e^2 = 6.6943799901413 $*10^{-3}$ [m]

c) Algorithm for Compensating Instrument Errors

The model of compensating the errors of IMU sensors is used in the algorithm in the form of [15]:

$$A_Z = M_{ZA} \cdot \left(K_A \cdot A_{RAW} - A_0 \right) \quad (5)$$

$$\omega_g = M_{GZ} \cdot \left(\omega_Z + M_\omega \cdot A_Z + \omega_0 + \omega_C \right) \quad (6)$$

Where:

A_Z is the vector of measured phantom acceleration used for velocity integration; A_{RAW} is the vector of accelerometer measurements, M_{ZA} the matrix of transition from accelerometer axes to an instrument trihedral; K_A is scale coefficients. A_0 is a zero drift. ω_g is a vector signal of platform control. ω_Z is a design angular velocity of platform control. M_{GZ} is a compensation matrix of couplings and scale factor errors of gyroscope torquers. M_ω is a matrix of dynamic drift coefficients; ω_0 is a zero drift vector (a drift constant component), and ω_C is a case drift vector.

d) Velocity Integration and Navigational Calculations

To obtain relative velocity in the reference CS, the following differential equation is integrated:

$$\dot{V}_s = a_s - (\omega_s^r + 2 \cdot U_s) \times V_s^r + g_s \quad (7)$$

where:

V_s^r is a vector of relative velocity in the reference Coordinate system. a_s is a vector of measured phantom acceleration, ω_s^r is a vector of relative rate of a reference Coordinate system, U_s is a vector of Earth angular velocity, and g_s is a gravity vector. The vector of relative velocity in the reference trihedral takes the form:

$$\vec{V}_s^r += \delta\vec{V}_s - (\vec{a}_s^k - \vec{g}) \cdot (t_m - t_{m-1}) \quad (8)$$

$$\vec{a}_s^k = (\vec{\omega}_s^r + 2 \cdot \vec{U}_s) \times \vec{V}_s^r$$

$\vec{g}$ vector projections in the fixed CS are $[0, 0, -g_T]$.

The module of normal specific gravity [15]:

$$g_T = g_e \cdot \left(1 + 0.005302 \sin^2 \varphi - 2\frac{h}{a} \right) \quad (9)$$

$$g_e = 9.780319 \text{ m/s}^2$$

$\vec{\omega}_s^r$ projections of the components of relative rate of reference trihedral on their proper axes have the following form[15]:

$$\begin{cases} \omega_{s1}^r = -k_{s2} \cdot V_{s2}^r + k_{s3} \cdot V_{s1}^r \\ \omega_{s2}^r = \ \ k_{s1} \cdot V_{s1}^r - k_{s3} \cdot V_{s2}^r \\ \omega_{s3}^r = \ \ k_E \cdot V_E \cdot tg\varphi \end{cases} \tag{10}$$

The curvature and pivoting of a flight path in the direction of the axes of the accompanying trihedral:

$$\begin{cases} k_{s1} = k_E \cos^2 A + k_N \sin^2 A \\ k_{s2} = k_E \sin^2 A + k_N \cos^2 A \\ k_{s3} = (k_N - k_E) \cdot \sin A \cdot \cos A \end{cases} \tag{11}$$

Where A is the azimuth angle from north direction

$$\begin{cases} k_E = \dfrac{1}{R_E} \\ k_N = k_E \cdot \dfrac{1 - e^2 \cdot \sin^2 \varphi}{1 - e^2} \\ R_E = \dfrac{a+h}{\sqrt{1 - e^2 \cdot \sin^2 \varphi}} \cong \dfrac{a}{\sqrt{1 - e^2 \cdot \sin^2 \varphi}} + h = N + h \end{cases} \tag{12}$$

The projections of Earth rate on the axes of the accompanying trihedral can be calculated by formulas [15]:

$$\begin{cases} U_{s1} = -U \cdot \cos\varphi \cdot \sin A = U \cdot C_{13}^{Nav} \\ U_{s2} = \ \ U \cdot \cos\varphi \cdot \cos A = U \cdot C_{23}^{Nav} \\ U_{s3} = \ \ U \cdot \sin\varphi \qquad\qquad = U \cdot C_{33}^{Nav} \end{cases} \tag{13}$$

where: U = 15.04107 [deg/h] = 7.292115 $\cdot 10^{-5}$ 1/s.

Absolute angular rate of the reference CS:

$$\vec{\omega}_s^a = \vec{\omega}_s^r + \vec{U}_s$$

The same vector in the axes of an instrument (fixed) trihedral:

$$\vec{\omega}_b^a = C^{Ori} \times \vec{\omega}_s^a$$

The components of a linear transition velocity of Earth rotation:

$$\begin{cases} V_{s1}^e = \ \ R_E \cdot U_{s2} \\ V_{s2}^e = -R_E \cdot U_{s1} \\ V_{s3}^e = \ \ 0 \end{cases} \tag{14}$$

The vector of absolute linear velocity in the reference trihedral:

$$\vec{V}_s^a = \vec{V}_s^r + \vec{V}_s^e .$$

The components of a relative linear velocity in the geographic trihedral:

$$\begin{cases} V_E^r = V_{s1}^r.\cos A + V_{s2}^r \cdot \sin A \\ V_N^r = -V_{s1}^r \cdot \sin A + V_{s2}^r \cdot \cos A \\ V_U^r = V_{s3}^r \end{cases} \tag{15}$$

e) Calculation Algorithm of Geographic Coordinates

Navigation algorithm is in the integration of $\dot{C}^{Nav} = \hat{\omega} \times C^{Nav}$ Poisson equation and thence finding the matrix of direction cosines of navigation trihedral orientation, and hence – geographic coordinates: φ (latitude), λ (longitude), A (azimuth) based on $C^{Nav}(0) = f(\varphi_0, \lambda_0, A_0)$ initial value and $\vec{\omega}$ angular rate of a navigation trihedral in the projections on its proper axes. Poisson equation is integrated on the basis of vector calculated value of $\vec{\omega}_s^r$ relative rate, and the matrix of direction cosines of navigation trihedral can be determined by formulas:

$$\dot{C}^{Nav} = \hat{\omega}_s^r \times C^{Nav}$$

$$\hat{\omega}_s^r = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} \tag{16}$$

$$C^{Nav}(t_0) = \begin{pmatrix} \sin\varphi\cdot\cos\lambda\cdot\sin A - \sin\lambda\cdot\cos A & \sin\varphi\cdot\sin\lambda\cdot\sin A + \cos\lambda\cdot\cos A & -\cos\varphi\cdot\sin A \\ -\sin\varphi\cdot\cos\lambda\cdot\cos A - \sin\lambda\cdot\sin A & \cos\lambda\cdot\sin A - \sin\varphi\cdot\sin\lambda\cdot\cos A & \cos\varphi\cdot\cos A \\ \cos\varphi\cdot\cos\lambda & \cos\varphi\cdot\sin\lambda & \sin\varphi \end{pmatrix} \tag{17}$$

$$\varphi = \arctan\frac{c_{33}}{\sqrt{c_{31}^2 + c_{32}^2}} \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]; \; \lambda = \arctan\frac{c_{32}}{c_{31}} \quad [-\pi, \pi]; \; A = \arctan\frac{-c_{13}}{c_{23}} \quad [-\pi, \pi]$$

4) In flight Alignment Error model

We introduce the error model, which appropriately describes the INS behavior during IFA. We then substantiate its adequacy for consideration as a piece-wise constant system (PWCS) and then the simpler stripped observability matrix (SOM) can be used in the system observability analysis. The most suitable model for IFA, where the measured states are the velocity states, is the, so called psi-model [11,12,13,14]. This model is in the form:

$$\begin{bmatrix} \delta\dot{V}_x \\ \delta\dot{V}_y \\ \delta\dot{V}_z \\ \dot{\psi}_x \\ \dot{\psi}_y \\ \dot{\psi}_z \end{bmatrix} = \begin{bmatrix} 0 & \tilde{\Omega}_U & -\tilde{\Omega}_N & 0 & -f_U & f_N \\ -\tilde{\Omega}_U & 0 & \tilde{\Omega}_E & f_U & 0 & -f_E \\ \tilde{\Omega}_N & -\tilde{\Omega}_E & 0 & -f_N & f_E & 0 \\ 0 & 0 & 0 & 0 & \Omega_U & -\Omega_N \\ 0 & 0 & 0 & -\Omega_U & 0 & \Omega_E \\ 0 & 0 & 0 & \Omega_N & -\Omega_E & 0 \end{bmatrix} \begin{bmatrix} \delta V_x \\ \delta V_y \\ \delta V_z \\ \psi_x \\ \psi_y \\ \psi_z \end{bmatrix} + \begin{bmatrix} \nabla_x \\ \nabla_y \\ \nabla_z \\ \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \tag{18}$$

Where δV and ψ represent the velocity error and attitude error states, respectively; ∇ and ε represent, respectively, the generalized accelerometer error and the generalized gyro drift rate; f_E, f_N , and f_U represent, respectively, the East, North, and Up components of the specific force sensed by INS respective accelerometers. λ and φ represent, respectively, the longitude and the latitude of the INS; and Ω is the earth rate .

$$\tilde{\Omega} = \begin{bmatrix} 0 & \tilde{\Omega}_U & -\tilde{\Omega}_N \\ -\tilde{\Omega}_U & 0 & \tilde{\Omega}_E \\ \tilde{\Omega}_N & -\tilde{\Omega}_E & 0 \end{bmatrix}; \; \Omega = \begin{bmatrix} 0 & \Omega_U & -\Omega_N \\ -\Omega_U & 0 & \Omega_E \\ \Omega_N & -\Omega_E & 0 \end{bmatrix}; F_j = \begin{bmatrix} 0 & -f_U & f_N \\ f_U & 0 & -f_E \\ -f_N & f_E & 0 \end{bmatrix} \tag{19}$$

Consequently, the carrier trajectory during the IFA maneuver can be characterized by a sequence of segments taken from table (1). The elements f_E, f_N, and f_U for each segment j are the entries of the sub matrices $\boldsymbol{F}_j$

Number of Segment (j)	Maneuver Characteristics	f_E	f_N	f_U
1	straight flight	0	0	-g
2	maneuver with north acceleration	0	f_N	-g
3	maneuver with east acceleration	f_E	0	-g
4	maneuver with horizontal acceleration	f_E	f_N	-g
5	Pull-up or dive	0	0	- f_U

Table (1) Trajectory segmentation

The observability analysis of the INS at segment j, after having gone through segments $1,2,...\mathrm{j}-1$ from table (1), the total observability matrix (TOM) $\tilde{Q}(j)$ is constructed as follows:

$$\tilde{Q}(j) = \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 e^{A_1\Delta_1} \\ \vdots \\ \tilde{Q}_j e^{A_{j-1}\Delta_{j-1}} \cdots e^{A_1\Delta_1} \end{bmatrix} \tag{20}$$

Where

$$\tilde{Q}_i^T = \left[H_i^T \quad (H_i A_i)^T \quad (H_i A_i^2)^T \quad \cdots \quad (H_i A_i^{n-1})^T \right] \; ,1 \le i \le j \tag{21}$$

Δ_i is the time span of segment i.

Consequently, instead of using the TOM of Equation (5.3.1) we may use $\tilde{Q}_s(j)$ stripped observability matrix (SOM), which simplifies the observability analysis considerably. The SOM $\tilde{Q}_s(j)$ is constructed as[14]:

$$\tilde{Q}_s(j) = \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \\ \vdots \\ \tilde{Q}_j \end{bmatrix} \qquad (22)$$

If $\text{null}(\tilde{Q}_j) \subset \text{null}(A_j)$, $\forall j \quad j = 1, 2 \ldots r$

Then, $\text{null}(\tilde{Q}_j) = \text{null}(\tilde{Q}_s)$; $\quad \text{rank}(\tilde{Q}_j) = \text{rank}(\tilde{Q}_s)$

5) Simulation and results

From the trajectory data, under the assumption of the heading angle $60[^o]$, constant velocity $400[\text{m/s}]$, initial misalignment angles is chosen as $1[^0]$, measuring error of velocity is 0.5 [m/s] , and the local latitude of SINS place is $30[^0]$. Also, the trajectory data, under the turn maneuver with maximum variation of heading angle $30[^0]$.

Table (2) shows the different convergence rates of ψ_x, ψ_y and ψ_z for 10-states and 12-states in the cases of straight level flight, and turn level maneuver. Also, figures (3, 4), and figures (5, 6) shows the result of azimuth estimated error, for 10-states and 12-states in the above two cases and figures (7,8,9,10) shows some of the missile flight parameters.

	Case	$\Delta\psi_x$ [sec]	$\Delta\psi_y$ [sec]	$\Delta\psi_z$ [sec]
10-states (2- channel)	Straight level-flight	**-6.3733**	**-29.1415**	**-211.3699**
	turn level maneuver	**0.8430**	**-0.5898**	**-1.7385**
12-states (3- channel)	Straight level-flight	**-6.4982**	**-28.8416**	**-231.7737**
	turn level maneuver	**-0.2757**	**0.3022**	**-3.7159**

Table (2) Estimation error of drift misalignment angles ψ_x, ψ_y and ψ_z for 10 and 12-states

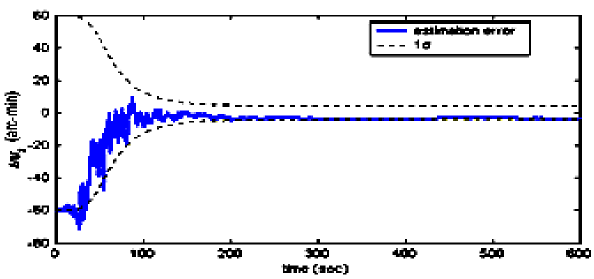


Fig (3) Azimuth error with Straight level-flight (10-states)

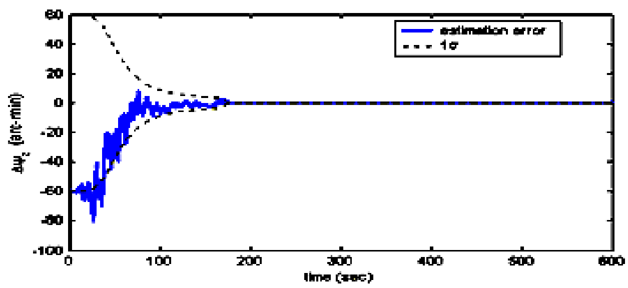
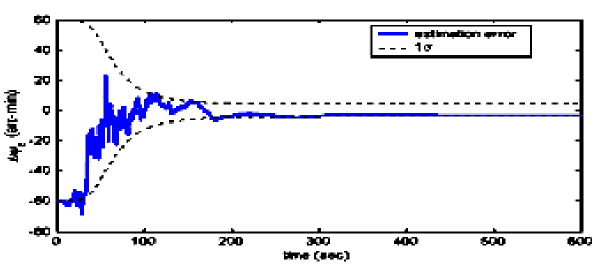
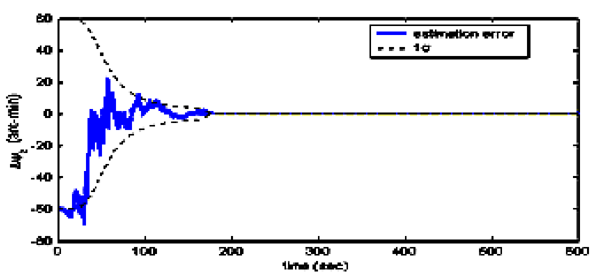


Fig (4) Azimuth error with turn level- maneuver (10-states)





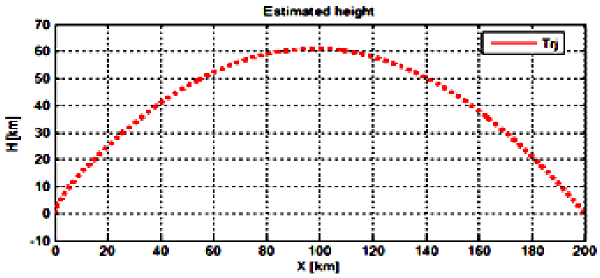


Fig (7) Missile Trajectory

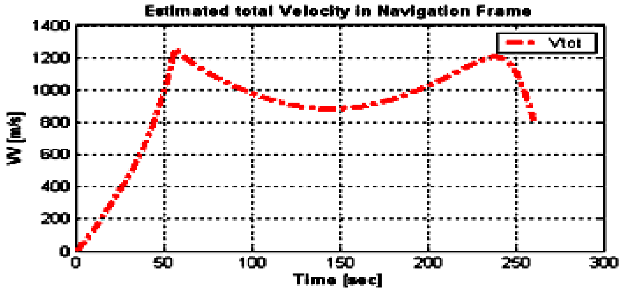


Fig (8) Missile Velocity

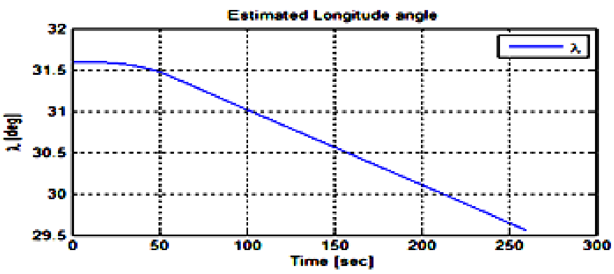


Fig (9) Missile Longitudinal

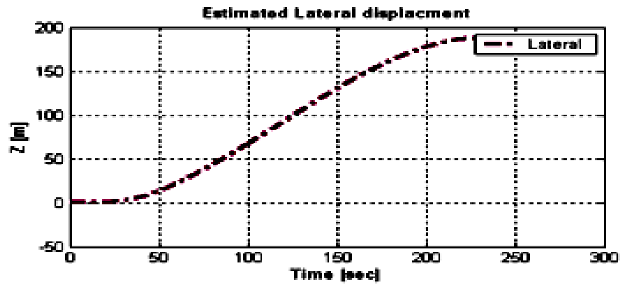


Fig (10) Missile Lateral displacement

Conclusion

The Launch vehicle will be controlled by a small on-board digital computer with navigation data from an inertial system. In this paper we focused on the Navigation mathematical Software (NMSW) which performs the following functions such as INS initial alignment, Solution of navigation problems: inertial velocity and coordinate reckoning in addition to inertial sensors calibration. The methods of testing the program are described, and a typical trajectory is given based on the hard ware in the loop missile flight simulation. The attention was focused on the derivation and programming of NMSW algorithm for SINS wanders azimuth mechanization. Based on the trajectory simulator data from the missile equation of motions the process of simulation and computation is highlighted.

Also, we introduced the error model, which appropriately describes the INS behavior during IFA. We then substantiated its adequacy for consideration as a piece-wise constant system (PWCS). Using PWCS we can simplify in-flight alignment observability analysis. The simulation results showed the effect of the variation of initial large azimuth misalignment angle for two-channel and three channels including the un-damped channel by using the extended Kalman filter (EKF).

References

[1]	Jong-Hyuk Kim, Stuart Wishart and Salah Sukkarieh, ‘’Real-time Navigation, Guidance, and Control of a UAV using Low-cost Sensors’’, Australian Centre for Field Robotics University of Sydney, NSW 2006, Australia.
[2]	Ali AKGÜL, Sartuk KARASOY ‘’ DEVELOPMENT OF A TACTICAL BALLISTIC MISSILE TRAJECTORY PREDICTION TOOL’’, ISTANBUL UNIVERSITY – JOURNAL OF ELECTRICAL & ELECTRONICS ENGINEERING,2005
[3]	B.L. Stevens, F.L. Lewis, ‘’Aircraft Control and Simulation’’, John Wiley and Sons, Inc., 1992.
[4]	Skogestad, Postlethwaite,‘’ Multivariable Feedback Design’’, John Wiley and Sons, Inc., 1997
[5]	Ibrahim I. Arafa, M. I. Latif, Zhong .H.Yue “ Correction of the Flight Path Based on Error Estimation Of SINS Ground Alignment Using Modern Computer Techniques” Beihang University, Beijing 100083,Conferences second Asian workshop of foundations of software. Nanjing China DEC. 2003
[6]	Ibrahim Ismail Arafa,’’ Analysis and Design of Guidance Navigation and Control Systems for Missile Applications’’, Beihang University Sep. 2004
[7]	Rodeny E. Stubbs, Whittak H. Huang and Eric Schimtz “ Integration of GPS into Ballistic Navigation Solution” Lockheed Martin Astronautics By American Institute of aeronaumatic and Astronautics, Inc. 1997
[8]	M. Napier, “Integration of Satellite and Inertial Positioning Systems” Institute of Engineering Surveying and Space Geodesy, University of Nottingham, Vol. 43, 1995
[9]	Eduardo, Durrant-Whyte Hugh. Department of Mechanical and Mechatronic Engineering University of Sydney, 2006, NSW, Australia, 0-8186- 8025-3/97IEEE 1997
[10]	Fuh Jiang Chung Shan Institute of Science and technology, Taiwan “Error Analysis of Analytic Coarse Alignment methods” IEEE Transaction on Aerospace and Electronic, 2. May 1997.
[11]	Kayton, M., and Fried, W.R.(1969) “Avionics Navigation Systems”
[12]	dle, J. R. (1983) “Inertial navigation system error-model considerations in Kalman filtering applications”. In C.T. Leondes (Ed.), Control and dynamics Systems. New York: Academic Press, 20 (1983)
[13]	Goshen-Meskin, Bar-Itzback Israel Aircraft and Institute of Technology “Observability Analysis of Piece-Wise Constant Systems-Part II: Application to inertial Navigation In-Flight Alignment” IEEE Transactions on aerospace and electronic systems Vol. 28, No. 4, October, 1992.
[14]	Goshen-Meskin, Bar-Itzback Israel Aircraft and Institute of Technology “Observability

	Analysis of Piece-Wise Constant Systems with Application to inertial Navigation" IEEE Transactions on aerospace and electronic systems Vol. 2, December 1990.
[15]	Mostafa elzoghoby M.S. Thesis "Error Estimation of INS Alignment on Stationary and Moving Bases through Observability Analysis" M.T.C. JULE 2011. Cairo, Egypt