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Strap down INS Alignment Using Non-Linear Model for Large Azimuth Misalignment

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Abstract

In this paper, a general non-linear psi-angle approach is presented that does not require coarse alignment. In the psi-angle model, The SINS error models (position, velocity and attitude angles) with large misalignment angles are derived. This error model uses Euler angles to describe the misalignment of platform frame to navigation frame, but the derived error models are only valid under condition that the heading uncertainty is large and the two leveling misalignment angles are small the three misalignment angles are assumed all large. We simulate SINS large azimuth misalignment angle on the stationary base (fixed position and multi-position) alignment also on the movable base (In-flight) alignment in two cases (straight level flight and turn level maneuver) based on the trajectory generator data. All these simulations are done using different Kalman filter techniques (nonlinear discrete equations, Standard extended Kalman filter EKF, and iterative filtering IKF) to solve the non-linear system problems.

Introduction

The alignment of a strap down inertial navigation system (SDINS) determines the transformation matrix between body frame and navigation frame in the local-level frame. The stationary initial alignment, which consists of a coarse alignment and a fine alignment, is usually performed when a vehicle is at rest. In this case, if low-grade sensors are used for cost reduction, it is virtually impossible to detect small attitude errors because its accuracy heavy depends on inertial sensors employed in alignments. For some applications, the coarse alignment is only performed or the initial attitude is directly obtained from other sources such as a stored attitude or a master inertial navigation system (INS) in order to reduce the initial alignment time. In cases mentioned above, the initial attitude errors may be very large. Large attitude errors do not guarantee the accuracy and reliability of a system after beginning a navigation mode. INS error propagation models for gimbal (GINS) and strap down inertial navigation units have been subject of significant research during the past few years [7], [5], [9], [4], [8], [6]. Two main approaches are used to derive these equations: psi-angle approach and perturbation approach [6], [9], and [7]. The INS alignment and calibration tasks are usually based on these models. In previous works the initial orientation errors are assumed to be small, i.e. less than 5 degrees. The system can then be approximated with linear models due to the small angle assumption. To satisfy these requirements good quality gyros and external tilt and heading information has to be used. So far, few works attempted to model large angle errors to consider, for example, large heading uncertainty of IMU orientation. In [11] and [12] an approximate extended psi-angle model with large heading misalignment is presented. It uses four states to describe the three-psi-angles. The model extension is very involved, and to the best of the author knowledge it has not been used in any practical application. Ref [10], introduced a Kalman filter mechanization for INS air start system. This approach uses two non-linear states to describe one heading angle. It still requires coarse ground alignment information within few degrees to estimate the wander angle. Ref [13] presents an INS error model considering large heading uncertainty and small tilt misalignment errors using a perturbation approach.

Non-linear Error Model for SINS

As we mentioned before, the objective of alignment process for a strap down is to determine the direction cosine matrix C_b^n which define the relationship between the inertial sensor axes and local geographic frame [5]. The measurements provided by the inertial sensors in bode axes may be resolved into the local geographic frame using the current best estimated of the body attitude w.r.t. this frame. The resolved sensor measurements are then compared with the expected turn rate and accelerations to enable the direction cosine to be calculated correctly. In the other word SINS uses mathematical platform rather than physical one (the mathematical platform is determined by C_b^n) we get the attitude error equations by disturbance of nominal equation, but for GINS we get the attitude error equations by using the actual error angles. Let, the misalignment angle ψ between platform frame and navigation frame is defined as:

$\psi = [\psi_x \quad \psi_y \quad \psi_z]^T$ and Ψ is the skew symmetric matrix of psi-angle.

Let, $s_x = \sin(\psi_x)$; $s_y = \sin(\psi_y)$; $s_z = \sin(\psi_z)$;
$c_x = \cos(\psi_x)$; $c_y = \cos(\psi_y)$; $c_z = \cos(\psi_z)$;

Then, the DCM from n-frame to p-frame can be defining as [7]:

$$C_n^p = \begin{bmatrix} c_y c_z - s_y s_x s_z & c_y s_z + s_y s_x c_z & -s_y c_x \\ -c_x s_z & c_x c_z & s_x \\ s_y c_z + c_y s_x s_z & s_y s_z - c_y s_x c_z & c_y c_x \end{bmatrix} \quad (1)$$

1) Velocity error model

The SINS true velocity error in navigation frame given by:

$$\dot{\boldsymbol{V}}_t^n = C_b^n \boldsymbol{f}^b - (2\Omega_{ie}^n + \omega_{en}^n) \times \boldsymbol{V}_t^n + \boldsymbol{g}^n \tag{2}$$

Where, $\boldsymbol{f}^b, C_b^n$ is the specific force in body frame and transformation matrix from body frame to navigation frame respectively. The SINS solves the following velocity $\boldsymbol{V}_c^c$ in the computational frame:

$$\dot{\boldsymbol{V}}_c^c = \hat{C}_b^n \hat{\boldsymbol{f}}^b - (2\hat{\Omega}_{ie}^n + \hat{\omega}_{en}^n) \times \boldsymbol{V}_c^c + \boldsymbol{g}^c \tag{3}$$

Where , $\hat{C}_b^n = C_b^p$.

$\boldsymbol{g}^c, \boldsymbol{g}^n$ is the gravity vector resolved in the computational frame and navigation frame respectively, $\mathbf{V}_c^c$ is the velocity vector resolved in the c-frame and can be calculated as:

$$\boldsymbol{V}_{\mathrm{c}}^{\mathrm{c}} = \boldsymbol{V}_{\mathrm{t}}^{\mathrm{n}} + \delta \boldsymbol{V}$$

$\hat{\mathbf{f}}^b$ is the estimated specific force in body frame that can be written as.

$$\hat{\boldsymbol{f}}^b = \boldsymbol{f}^b + \nabla^b$$

Where, ∇^b is the specific force error due to accelerometer bias in (b-frame). And

$$\hat{\Omega}_{ie}^n = \Omega_{ie}^n + \delta\Omega_{ie}^n$$
$$\hat{\omega}_{en}^n = \omega_{en}^n + \delta\omega_{en}^n$$

Subtract Equ. (2) from Equ. (3) yields the SINS velocity error equation:

$$\begin{aligned}\delta\dot{\boldsymbol{V}} = &(\mathbf{C}_n^p - \boldsymbol{I})\mathbf{C}_b^n \boldsymbol{f}^b + \hat{\mathbf{C}}_b^n \nabla^b - (2\delta\Omega_{ie}^n + \delta\omega_{en}^n) \\ &\times \boldsymbol{V}_t^n - (2\Omega_{ie}^n + \omega_{en}^n) \times \delta\boldsymbol{V} + \delta\boldsymbol{g}^n\end{aligned} \tag{4}$$

The above equation as a function of estimated specific force $\hat{\mathbf{f}}^b$ could be written as:

$$\begin{aligned}\delta\dot{\boldsymbol{V}} = &(\boldsymbol{I} - C_p^n)\hat{C}_b^n \hat{\boldsymbol{f}}^b + C_b^n \nabla^b - (2\delta\Omega_{ie}^n + \delta\omega_{en}^n) \\ &\times \boldsymbol{V}_t^n - (2\Omega_{ie}^n + \omega_{en}^n) \times \delta\boldsymbol{V} + \delta\boldsymbol{g}^n\end{aligned} \tag{5}$$

Let $\hat{C}_b^n \hat{\boldsymbol{f}}^b = \hat{\boldsymbol{f}}^p$, $C_{\mathrm{b}}^{\mathrm{n}} \nabla^b = C_p^n \nabla^p$ and when the attitude error is small then, the transformation matrix C_n^p can be expresses as:

$$C_n^p = \boldsymbol{I} - [\Phi \times] \tag{6}$$

when the attitude error is small, as we know from Equ. (6) the small disturbance equations can be written as follows:

$$\delta\dot{V} = (C_b^n f^b)\times\Phi - (2\delta\Omega_{ie}^n + \delta\omega_{en}^n) \times V_t^n - (2\Omega_{ie}^n + \omega_{en}^n)\times\delta V + \hat{C}_b^n \nabla^b + \delta g^n \tag{7}$$

2) Attitude Error Model

The psi-angle model for small angle errors was presented before in [1] and [2]. This paper presents a new psi- angle model that can be used with large angle errors.

The true transformation matrix C_b^n can be written as:

$$\dot{C}_b^n = C_b^n[\omega_{ib}^b\times] - [\omega_{in}^n\times]C_b^n \tag{8}$$

Where, $[\omega_{ib}^b\times]$ is the skew symmetric matrix and ω_{ib}^b is the computed angular velocity of body w.r.t. inertial frame.

The matrix C_b^p is obtained using measured gyro rates $\hat{\omega}_{ib}^b$ provided by the IMU:

$$\dot{C}_b^p = C_b^p[\hat{\omega}_{ib}^b\times] - [\hat{\omega}_{in}^n\times]C_b^p \tag{9}$$

Where, C_b^p is the transformation matrix from body to platform frame (or written as $\hat{C}_b^n$); $\hat{\omega}_{in}^n = \omega_{in}^n + \delta\omega_{in}^n$ where $\omega_{in}^n, \delta\omega_{in}^n$ is the true angular velocity and angular velocity error of navigation frame w.r.t. inertial frame. $\hat{\omega}_{ib}^b$ Contains gyros drift errors $\delta\omega_{ib}^b$ that can be large, especially when working with low cost IMU:

$$\hat{\omega}_{ib}^b = \omega_{ib}^b + \delta\omega_{ib}^b$$

Let $\Delta C = C_b^p - C_b^n$, then

$$\Delta C = C_b^p - C_b^n = C_b^p - C_p^n C_b^p = (I - C_p^n)C_b^p \tag{10}$$

$\Delta\dot{C}$ can be derived from Equ. (9,10):

$$\Delta\dot{C} = (I - C_p^n)\dot{C}_b^p - \dot{C}_p^n C_b^p = C_b^p[\hat{\omega}_{ib}^b\times] - [\hat{\omega}_{in}^n\times]C_b^p - C_p^n C_b^p[\hat{\omega}_{ib}^b\times] + C_p^n[\hat{\omega}_{in}^n\times]C_p^n C_b^p - \dot{C}_p^n C_b^p \tag{11}$$

$\Delta\dot{C}$ can also obtained from $\Delta C = C_b^p - C_b^n$

$$\Delta\dot{C} = \dot{C}_b^p - \dot{C}_b^n = C_b^p[\hat{\omega}_{ib}^b\times] - [\hat{\omega}_{in}^n\times]C_b^n - C_p^n C_b^p[\omega_{ib}^b\times] - [\omega_{in}^n\times]C_p^n C_b^p \tag{12}$$

From Equ. (11,12), we get:

$$C_p^n C_b^p \left[\delta\omega_{ib}^b \times\right] - C_p^n \left[\hat{\omega}_{in}^n \times\right] C_b^p + \dot{C}_p^n C_b^p + \left[\omega_{in}^n \times\right] C_p^n C_b^p = 0 \tag{13}$$

Right multiply C_p^b to the above equation yield:

$$\dot{C}_p^n + C_p^n C_b^p \left[\delta\omega_{ib}^b \times\right] C_p^b - C_p^n \left[\hat{\omega}_{in}^n \times\right] + \left[\omega_{in}^n \times\right] C_p^n = 0 \tag{14}$$

It can proved that

$$\left[\delta\omega_{ib}^p \times\right] = C_b^p \left[\delta\omega_{ib}^b \times\right] C_p^b = C_n^p \left[\delta\omega_{ib}^n \times\right] C_p^n \tag{15}$$

And $C_p^n \left[\delta\omega_{ib}^p \times\right] = \left[\delta\omega_{ib}^n \times\right] C_p^n$. Consequently, Equ. (15) can be simplified as:

$$\dot{C}_p^n + \left[\delta\omega_{ib}^n \times\right] C_p^n - C_p^n \left[\hat{\omega}_{in}^n \times\right] + \left[\omega_{in}^n \times\right] C_p^n = 0 \tag{16}$$

Replacing $\dot{C}_p^n = C_p^n \left[\omega_{np}^p \times\right]$, and left multiplying C_n^p to Equ. (16):

$$\left[\omega_{np}^p \times\right] + C_n^p \left[\delta\omega_{ib}^n \times\right] C_p^n - \left[\hat{\omega}_{in}^n \times\right] + C_n^p \left[\omega_{in}^n \times\right] C_p^n = 0 \tag{17}$$

We can notice that: $C_n^p \left[\delta\omega_{ib}^n \times\right] C_p^n = \left[\delta\omega_{ib}^p \times\right]$ and $C_n^p \left[\omega_{in}^n \times\right] C_p^n = \left[\omega_{in}^p \times\right]$, then Equ. (17) changes to:

$$\left[\omega_{np}^p \times\right] + \left[\delta\omega_{ib}^p \times\right] - \left[\hat{\omega}_{in}^n \times\right] + \left[\omega_{in}^p \times\right] = 0 \tag{18}$$

$\left[\omega_{np}^p \times\right] + \left[\delta\omega_{ib}^p \times\right] - \left[\hat{\omega}_{in}^n \times\right] + \left[\omega_{in}^p \times\right]$ are the sums of skew symmetric matrices of $\left(\omega_{np}^p + \delta\omega_{ib}^p - \hat{\omega}_{in}^n + \omega_{in}^p\right)$, then Equ. (18) can be written in the following form:

$$\omega_{np}^p + \delta\omega_{ib}^p - \hat{\omega}_{in}^n + \omega_{in}^p = 0 \tag{19}$$

with $\omega_{in}^p = C_n^p \omega_{in}^n$ and $\hat{\omega}_{in}^n = \omega_{in}^n + \delta\omega_{in}^n$ then Equ. (19) can be written as:

$$\omega_{np}^p = \omega_{in}^n + \delta\omega_{in}^n - C_n^p \omega_{in}^n - C_b^p \delta\omega_{ib}^b = \left(\boldsymbol{I} - C_n^p\right)\omega_{in}^n + \delta\omega_{in}^n - C_b^p \delta\omega_{ib}^b \tag{20}$$

The three components of the Euler angle $\dot{\psi}$ is not orthogonal so that, the relation between $\dot{\psi}$ and ω_{np}^p can be written as [7]:

$$\omega_{np}^{p} = C_n^p \begin{bmatrix} 0 \\ 0 \\ \dot{\psi}_z \end{bmatrix} + \begin{bmatrix} c_y & 0 & -s_y \\ 0 & 1 & 0 \\ s_y & 0 & c_y \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_x & s_x \\ 0 & -s_y & c_x \end{bmatrix} \begin{bmatrix} \dot{\psi}_x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} c_y & 0 & -s_y \\ 0 & 1 & 0 \\ s_y & 0 & c_y \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\psi}_y \\ 0 \end{bmatrix} \tag{21}$$

Using Equ. (1) yields:

$$\omega_{np}^{p} = \begin{bmatrix} \dot{\psi}_x c_y - \dot{\psi}_z s_y c_x \\ \dot{\psi}_y + \dot{\psi}_z s_x \\ \dot{\psi}_x s_y + \dot{\psi}_z c_y c_x \end{bmatrix} \tag{22}$$

In alignment, if we consider $\dot{\psi}$ is small and if the horizontal misalignment tilt angle ψ_x, ψ_y is also small then, the angular velocity of platform w.r.t. navigation frame will be as:

$$\omega_{np}^{p} \approx \dot{\psi} = \begin{bmatrix} \dot{\psi}_x \\ \dot{\psi}_y \\ \dot{\psi}_z \end{bmatrix} \tag{23}$$

When the horizontal misalignment angle ψ_x, ψ_y is small as we know from Equ. (23) the SINS attitude error equation can be written as:

$$\dot{\psi} \approx \omega_{np}^{p} = \left(\boldsymbol{I} - C_n^p\right)\omega_{in}^{n} + \delta\omega_{in}^{n} - C_b^p \delta\omega_{ib}^{b} \tag{24}$$

Then, Equ. (24) is the general psi-angle error model that can be used for small or large angle errors for SINS. When the three misalignment angles are small, then the attitude error model using Euler angle can be simplified to Φ angle as:

$$\dot{\Phi} = \Phi \times \omega_{in}^{n} + \delta\omega_{in}^{n} - C_b^p \delta\omega_{ib}^{b} \tag{25}$$

Stationary Base Alignment Mode

In the stationary alignment we usually suppose that the position is known and fixed. So, the values of velocity vector $\boldsymbol{V}_t^n$ and angular velocity vector ω_{en}^{n} equal zero. Then Equ. (7,24) can be written as:

$$\delta\dot{\boldsymbol{V}} = (C_n^p - \boldsymbol{I})\, C_b^n\, \boldsymbol{f}^{\boldsymbol{b}} + \hat{C}_b^n \nabla^{\boldsymbol{b}} - \delta\omega_{\boldsymbol{en}}^{\boldsymbol{n}} \times \boldsymbol{V}_t^n - 2\Omega_{ie}^{n} \times \delta\boldsymbol{V} \tag{26}$$

$$\dot{\psi} \approx \omega_{np}^{p} = \left(\boldsymbol{I} - C_n^p\right)\Omega_{ie}^{n} + \delta\omega_{in}^{n} - C_b^p \delta\omega_{ib}^{b} \tag{27}$$

Where $\omega_{\boldsymbol{in}}^{\boldsymbol{n}} = \omega_{\boldsymbol{en}}^{\boldsymbol{n}} + \Omega_{\boldsymbol{ie}}^{\boldsymbol{n}}$

The value of angular velocity of the earth and the specific forces in navigation frame can be written as:

$$\Omega_{ie}^{n} = \begin{bmatrix} 0 & \Omega\cos\phi & \Omega\sin\phi \end{bmatrix}^{T} \tag{28}$$

$$\boldsymbol{f}^{\,n} = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^{T} \tag{29}$$

Substituting Equ. (28,29) into Equ. (26,27) then the platform error model on the stationary base can be written as:

$$\begin{aligned} \delta\dot{V}_x &= -\psi_y g + 2\Omega\sin\phi\,\delta V_y + \nabla_x^{p} \\ \delta\dot{V}_y &= \psi_x g - 2\Omega\sin\phi\,\delta V_x + \nabla_y^{p} \end{aligned} \tag{30}$$

$$\begin{aligned} \dot{\psi}_x &= -\sin\psi_z\,\Omega\cos\phi + \psi_y\Omega\sin\phi - \delta V_y/(R_M + h) + \varepsilon_x^{p} \\ \dot{\psi}_y &= (1-\cos\psi_z)\Omega\cos\phi - \psi_x\Omega\sin\phi + \delta V_x/(R_N + h) + \varepsilon_y^{p} \\ \dot{\psi}_z &= (\psi_x\cos\psi_z - \psi_y\sin\psi_z)\Omega\cos\phi + \delta V_x\tan\phi/(R_N + h) + \varepsilon_z^{p} \end{aligned} \tag{31}$$

In-Flight Alignment Mode

In-flight alignment we usually get the INS velocity error equation from the acceleration measurement $\hat{\boldsymbol{f}}^{\,p}$ then, from Equ. (5) we get:

$$\delta\dot{\boldsymbol{V}} = (\boldsymbol{I} - C_p^n)\hat{C}_b^n\hat{\boldsymbol{f}}^{\,\boldsymbol{b}} + C_b^n\nabla^{\boldsymbol{b}} - (2\delta\Omega_{ie}^n + \delta\omega_{en}^n)\times\boldsymbol{V}_t^n - (2\Omega_{ie}^n + \omega_{en}^n)\times\delta\boldsymbol{V} + \delta\boldsymbol{g}^n$$

The error model in-flight augmented with sensor errors can be written as:

$$\begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}(\boldsymbol{t}) & \boldsymbol{I}_{6\times6} \\ 0_{6\times6} & 0_{6\times6} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{q}(\boldsymbol{x}, t) \\ 0_{6\times1} \end{bmatrix} \tag{32}$$

A local level ENU (East-North-Up) frame is used as the navigation frame, vertical channel included. The state vectors consists of:

$$\begin{aligned} \boldsymbol{x}_1 &= \begin{bmatrix} \delta V_{\boldsymbol{x}} & \delta V_{\boldsymbol{y}} & \delta V_{\boldsymbol{z}} & \psi_{\boldsymbol{x}} & \psi_{\boldsymbol{y}} & \psi_{\boldsymbol{z}} \end{bmatrix} \\ \boldsymbol{x}_2 &= \begin{bmatrix} \nabla_{\boldsymbol{x}} & \nabla_{\boldsymbol{y}} & \nabla_{\boldsymbol{z}} & \varepsilon_x^p & \varepsilon_y^p & \varepsilon_z^p \end{bmatrix} \end{aligned} \tag{33}$$

The linear part coefficient matrix $\boldsymbol{A}(t)$ is the system dynamic matrix defined as [9]:

$\mathbf{q}(\mathbf{x}, \boldsymbol{t})$ is the nonlinear part and can be computed as:

$$\boldsymbol{q}(\boldsymbol{x}, t) = \begin{bmatrix} (\boldsymbol{I} - C_p^n)\hat{\boldsymbol{f}}^{\,p} \\ (\boldsymbol{I} - C_n^p)\omega_{in}^{n} \end{bmatrix} \tag{34}$$

System Jacobian matrix can be computed as:

$$\frac{\partial \boldsymbol{f}(\boldsymbol{x},\mathrm{t})}{\partial \boldsymbol{x}} = \boldsymbol{A}(\mathrm{t}) + \frac{\partial \boldsymbol{q}(\boldsymbol{x},\mathrm{t})}{\partial \boldsymbol{x}} \tag{35}$$

Filtering Mechanization
In this section we simulate SINS large azimuth misalignment angle on the stationary base, in-flight alignment and multi-position alignment by using nonlinear discretization, Extended KF and iterative filtering.

Nonlinear discretization

To be convenient, we write the nonlinear system states equation as:

$$\dot{\boldsymbol{x}}(\mathrm{t}) = \boldsymbol{f}(\boldsymbol{x},\mathrm{t}) + \mathbf{w}(\mathrm{t}) \tag{36}$$

Where $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{t})$ is the nonlinear function, $\mathrm{w}(\mathrm{t})$ is the process noise. The variant matrix:

$$\mathbf{Q}(\mathrm{t}) = \boldsymbol{E}\left[\mathbf{w}(\mathrm{t})\mathbf{w}^{T}(\mathrm{t})\right]$$

Let $\Delta\mathrm{t}$ is the sampling time, and the solution of the scalar differential equation using Taylor expansion of $\boldsymbol{x}(\mathrm{t}+\Delta\mathrm{t})$:

$$\boldsymbol{x}(\mathrm{t}+\Delta\mathrm{t}) = \boldsymbol{x}(\mathrm{t}) + \boldsymbol{f}[\boldsymbol{x}(\mathrm{t})]\Delta\mathrm{t} + \frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}}\Big|_{\mathrm{x}=\mathrm{x}(t)} \boldsymbol{f}[\boldsymbol{x}(\mathrm{t})]\frac{(\Delta\mathrm{t})^2}{2} + \ldots.. \tag{37}$$

Let $\boldsymbol{x}_{\boldsymbol{k}} = \boldsymbol{x}(\mathrm{t})$, $\boldsymbol{x}_{\mathrm{k}+1} = \boldsymbol{x}(\mathrm{t}+\Delta\mathrm{t})$, ignore 2nd order derivative term, the discretization equation can be written as:

$$\boldsymbol{x}_{\boldsymbol{k}+1} = \boldsymbol{x}_{\boldsymbol{k}} + \boldsymbol{f}(\boldsymbol{x}_{\boldsymbol{k}})\Delta\mathrm{t} + \boldsymbol{D}(\boldsymbol{x}_{\boldsymbol{k}})\boldsymbol{f}(\boldsymbol{x}_{\boldsymbol{k}})\frac{(\Delta\mathrm{t})^2}{2} + \mathrm{w}_{\boldsymbol{k}} \tag{38}$$

Where

$$\boldsymbol{D}(\boldsymbol{x}_{\boldsymbol{k}}) = \frac{\partial \boldsymbol{f}(\boldsymbol{x},\mathrm{t})}{\partial \boldsymbol{x}}\Big|_{\mathrm{x}=\mathrm{x}_{\boldsymbol{k}}}$$

$\mathrm{w}_{\boldsymbol{k}}$ is the discretization process noise, and variant matrix $\boldsymbol{Q}_{\boldsymbol{K}} = \mathrm{E}\left[\mathrm{w}_{\boldsymbol{k}}\mathrm{w}_{\boldsymbol{k}}^{\boldsymbol{T}}\right]$

$$\boldsymbol{Q}_{\boldsymbol{k}} = \boldsymbol{Q}(\mathrm{t}).\Delta\mathrm{t} \tag{39}$$

We use the two horizontal velocity error measurements, and then the measurement equation can be written as:

$$z_k = H_k x_k + v_k \tag{40}$$

Where v_k is the measurement noise and measurement matrix can be written as:

$$H = [I_{2\times2} \vdots 0_{2\times8}] \tag{41}$$

Extended Kalman Filter (EKF)

To use extended Kalman Filter we must linearize the nonlinear equation because the measurement equation is linear [2]. So, we only need linearized system equation. Let Equ.(38) rewrite as:

$$x_{k+1} = F[x_k, k] + w_k \tag{42}$$

Where

$$F[x_k, k] = x_k + f(x_k)\Delta t + D(x_k) f(x_k) \frac{\Delta t^2}{2} \tag{43}$$

Let Δt is small, the transition matrix with one step can be written as:

$$\Phi(k+1, k) = \frac{\partial F[x(k), k]}{\partial x}\Big|_{x=\hat{x}(k/k)} \approx I + D[\hat{x}(k/k]\Delta t \tag{44}$$

So, the extended Kalman Filter can be programmed as:

$$\hat{x}_{k+1/k} = \hat{x}_{k/k} + f[\hat{x}_{k/k}]\Delta t + D[\hat{x}_{k/k}] f[\hat{x}_{k/k}]\Delta t^2/2$$

$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}[z_{k+1} - H_{k+1}\hat{x}_{k+1/k}]$$

$$P(k+1/k) = \Phi(k+1, k)P(k/k)\Phi^T(k+1, k) + Q_k$$

$$K_{k+1} = P_{k+1/k} H_{k+1}^T [H_{k+1} P_{k+1/k} H_{k+1}^T + R_{k+1}]^{-1}$$

$$P_{k+1/k+1} = [I - K_{k+1} H_{k+1}] P_{k+1/k} [I - K_{k+1} H_{k+1}]^T + K_{k+1} R_{k+1} K_{k+1}^T$$

Iterative filtering (IKF)

$$\begin{aligned} \hat{x}_{k/k+1} &= \hat{x}_{k/k} + P_{k/k}\Phi_{k+1/k}^T P_{k+1/k}^{-1}[\hat{x}_{k+1/k+1} - \hat{x}_{k/k+1}] \\ &= \hat{x}_{k/k} + P_{k/k}\Phi_{k+1/k}^T H_k^T [H_{k+1} P_{k+1/k} H_{k+1}^T + R_{k+1}]^{-1} \\ &\quad [z_{k+1} - H_{k+1}\hat{x}_{k+1/k}] \end{aligned} \tag{45}$$

The transition matrix with one step can be written as:

$$\Phi(k+1, k) = \frac{\partial F[x(k), k]}{\partial x}\Big|_{x=\hat{x}(k/k+1)} \approx I + D[\hat{x}(k/k+1]\Delta t$$

Computer Simulation

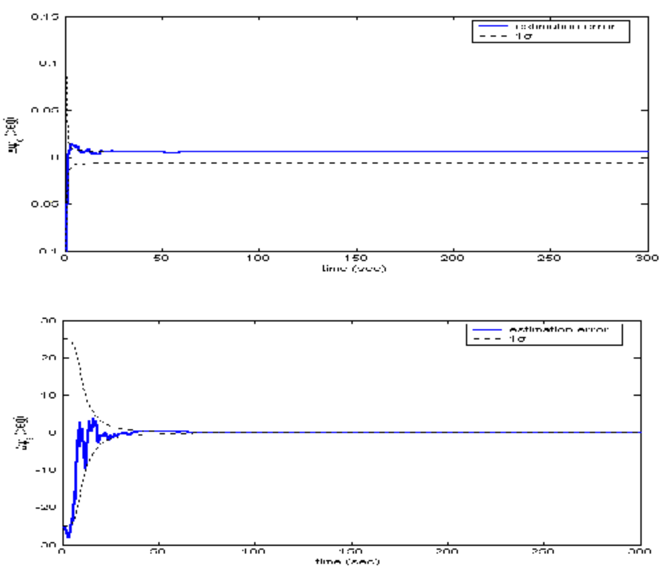
To be convenient to compare with result using small disturbance equation we use computation to evaluate the alignment accuracy. The initial attitude angles are chosen equal zero. The constant and random biases of each accelerometer are chosen as $100\mu g$ and $5\mu g$ respectively, and the constant and random drifts of each gyro are chosen as $0.02^0/h$ and $0.01^0/h$ respectively. The measuring error of velocity is $0.1\,\text{m/s}$ and the system measured noise $\sigma_{\delta V} = .01m/s$. The local latitude of SINS place is 30^0 The sampling time chosen as $50\,\text{msec}$.

Stationary alignment simulation

The initial pitch and roll angle errors chosen as 1^0, yaw error equal 20^0, we used nonlinear error equation and small disturbance equation separately, alignment time equal $300\,\text{sec}$. We used nonlinear model and EKF, then the estimated error of the horizontal error angle and azimuth error angle are shown in Fig (1), and Fig (2) shown the estimated accelerometer biases and gyro drift. Table (1) shown the effect of the variation of initial large azimuth misalignment angle on the static attitude error by using the EKF, IKF, and linearized Kalman filter (LKF).

Initial Azimuth [deg]	Est. Error angle [sec]	EKF	IKF	LKF
10	$\Delta\psi$	-254.2412	-244.0982	215.1588
	$\Delta\theta$	-21.2108	-21.3302	-255.0105
	$\Delta\gamma$	15.7242	15.7585	150.8116
15	$\Delta\psi$	-223.7322	-210.7751	1420.700
	$\Delta\theta$	-23.0430	-23.1915	-394.900
	$\Delta\gamma$	14.6770	14.6699	138.300
20	$\Delta\psi$	-191.1677	-176.0421	366.150
	$\Delta\theta$	-24.7919	-24.9570	-559.700
	$\Delta\gamma$	13.2539	13.1829	66.600

Table (1) Effect of the variation of initial large azimuth misalignment



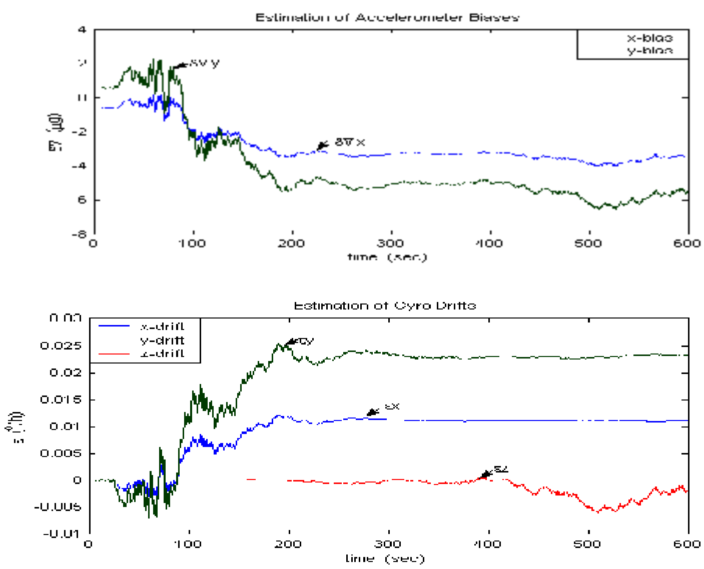


Fig (1) Estimated Attitude error angles **Fig (2) Estimated sensor bias and drift**

In-flight alignment Simulation

From the From the trajectory data, under the assumption firstly, the missile flight is linear and secondly, with horizontal level maneuver with max heading variation chosen as 30^0. The missile constant velocity $400\,\mathrm{m/s}$, flight direction angle chosen as 60^o, and the local latitude of SINS place is 30^0. The initial pitch, roll angle errors chosen as 1^0, and heading error chosen as 10^0. The total time of alignment equal $600\,\mathrm{sec}$. We used nonlinear error equation and small disturbance equation for in-flight alignment. Table (2) shown the static values of attitude error angles by using the different model of Kalman filters (EKF, IKF and LKF) in the case of linear flight and with turn maneuver.

Fig (3) shows the estimated errors of the horizontal error angle and azimuth error angle in the case of linear flight path. Fig (4) shows the estimated errors of the horizontal error angle and azimuth error angle in the turn maneuver. Fig (5,6) shows the static values of attitude error angles in the linear flight and with level maneuver using EKF respectively.

In-flight Cases	Attitude error [sec]	EKF	IKF	LKF
Linear Flight	$\Delta\psi$	-185.319	-140.9032	374.336
	$\Delta\theta$	-22.8078	-23.8795	-52.2318
	$\Delta\gamma$	15.2366	15.0149	230.385
Turn Maneuver	$\Delta\psi$	-0.0633	-0.6133	2164.6
	$\Delta\theta$	-0.1736	-0.2888	501.400
	$\Delta\gamma$	-0.0040	-0.1224	241.700

Table (2) Static Attitude Error Angle

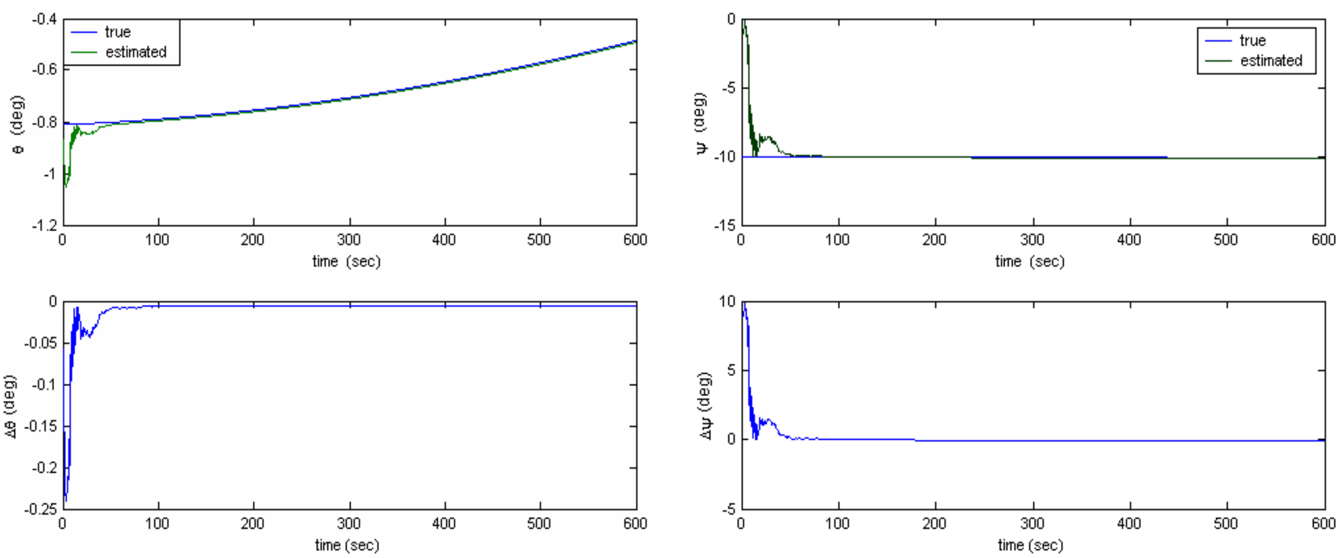


Fig (6,7) Estimated yaw and pitch error angles with Linear-flight

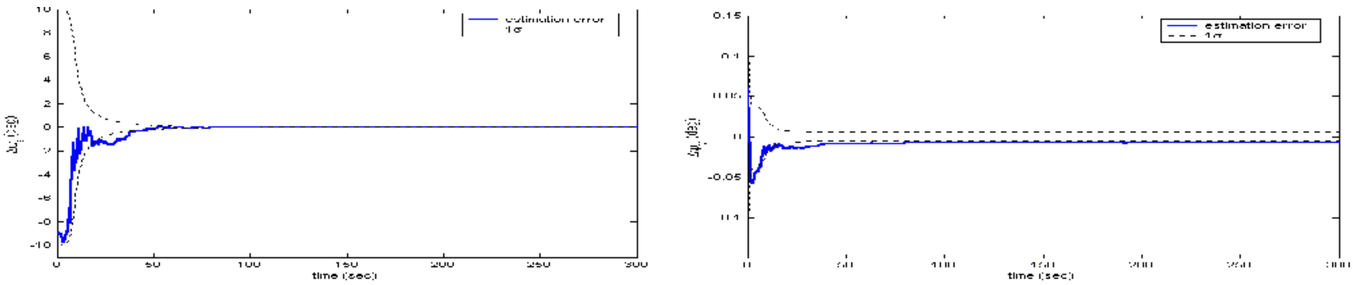


Fig (3) Estimated horizontal and azimuth error angle with linear flight

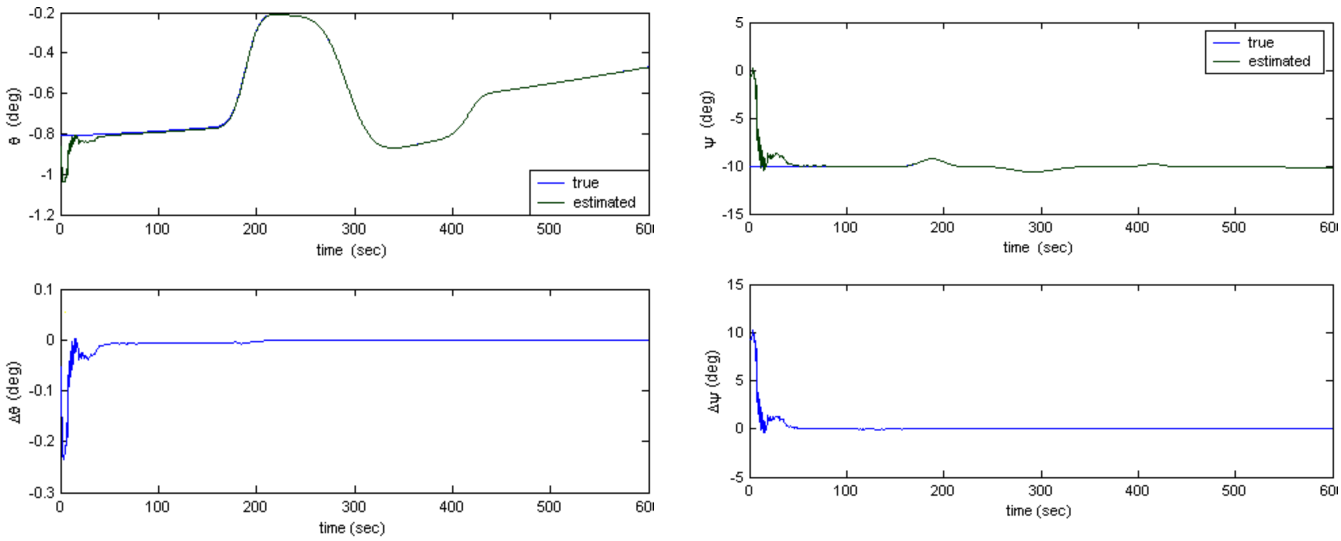


Fig (4,5) Estimated yaw and pitch error angles in turn maneuver

Conclusion

This paper presents a general nonlinear psi angles model that does not require coarse alignment. In this model, the azimuth misalignment angle is assumed large. The velocity error model is also presented. In this paper some SDINS error model which can be used to design an alignment filter. Three different Kalman filter are designed based on the SINS nonlinear error model. The model presents was validated with a set of experimental results of stationary alignment and In-flight alignment using kinematics trajectory data to estimate all the parameters of inertial navigation system needed for the alignment and calibration. These results are helpful in design of stationary alignment process to improve the performance of the INS alignment during In-flight mode.

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