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Hybrid FrFT and FFT based Multimode Transmission OFDM System Based

By

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Abstract:

The popularity of multicarrier systems such as orthogonal frequency division multiplexing (OFDM) is based on the ability to cancel inter-symbol-interference (ISI) and channel distortion using a single tap equalizer in the case of a stationary channel. In doubly selective fading channels OFDM transceivers are unable to diagonalize the channel matrix and consequently inter carrier interference (ICI) appears. The discrete fractional Fourier transform (FrFT) has been suggested to enhance performance over traditional OFDM systems when transmitting over doubly-dispersive channels. In this paper a novel hybrid multicarrier system based on the Fractional Fourier transform and Fourier transform is presented. Taking advantage of these properties, we propose an adaptive transmission in those channels using multimode transmission of FrFT-OFDM and FT-OFDM, and evaluate its performances through computer simulations.

Keywords:

Discrete Fourier transform, discrete fractional Fourier transform, Mobile Digital video, Digital Video Broadcasting, Orthogonal frequency-division multiplexing, Time-varying frequency-selective channels.

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1. Introduction:

In Fractional Fourier Transform based OFDM systems (FrFT-OFDM) the traditional Fourier transform is replaced by the fractional Fourier transform to modulate and demodulate the symbols[1]. The FrFT-OFDM was shown to provide better BER performance in doubly selective channels (which is time and frequency selective) that cannot be achieved with conventional multiplexing methods such as TDM (Time division multiplexing) or OFDM even with complex equalizers[1-3]. Superior BER implies better quality especially for moving receivers. On the other hand, in multipath fading environment the OFDM can diagonalize the channel matrix and mitigate the channel effect on the received symbols by a single equaliser coefficient per carrier. The OFDM has an acceptable BER performance in multipath fading environments with a resulting complexity that is less than the FrFT-OFDM [1, 4-6]. This lower signal processing complexity leads to less power consumption. The last two properties are important concepts in the Digital Video Broadcasting-Handheld (DVB-H) receivers which involve battery operated moving receivers as it needs low power consumption and acceptable BER to provide long life battery and a good quality reception while moving.

In this paper a multimode transmission system is proposed that takes advantage of these properties. We present a smart multimode transmission method using Multicarrier Transceivers based on Fractional Fourier Transform or Fourier Transform Based OFDM systems. In the case of multipath fading the conventional OFDM mode is used to benefit from the low complexity and the acceptable BER while in the case of doubly selective channel the FrFT-OFDM mode is used to benefit from the good BER. The receiver is able to feedback the mode to the transmitter using its knowledge of the channel status and the receiver speed (Doppler frequency). We evaluate the new Multimode Transmission system performances through computer simulations in various operational scenarios.

The remainder of the paper is organized as follows. In Section 2, the Fourier transform and its related OFDM system model is introduced. In Section 3, the Fractional Fourier transform and its related multicarrier system model is introduced. In Section 4, the multimode transmission system model is presented and description of the key realization of the transceiver is provided. In Section 5, equalization of both FrFT-OFDM and OFDM systems is considered. In Section 6, experiment and evaluation results are provided that compares the performance of the proposed multimode system to that of the traditional OFDM system and the FrFT-OFDM. The paper is concluded in section 7.

2. The Fourier Transform System Model

a. The Fourier Transform

The well-known Fourier transform is a mathematical operation that decomposes a function into its constituent frequencies. Which mean transforming a function from the time domain to the frequency domain. The Fourier transform is given by:

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi x\xi}\, dx \qquad (1)$$

and the inverse transformation is given by:

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{j2\pi x\xi}\, d\xi \qquad (2)$$

where $\hat{f}(\xi)$ is the Fourier transform of the function $f(x)$.

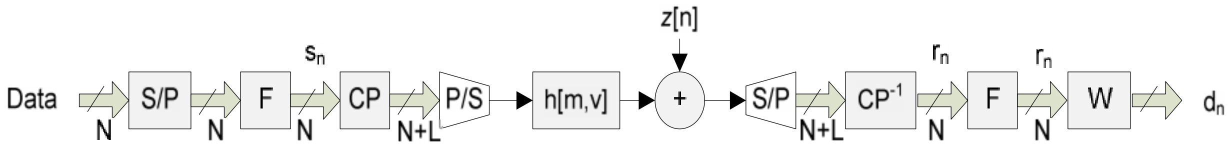

Fig.1 *The OFDM system*

b. The OFDM system

The OFDM system is shown in Fig.1.

Let $\{s_{n,k}\}_{k=0}^{N-1}$ with $E|s_{n,k}|^2 = \sigma_s^2$ be the complex symbols to be transmitted at the nthOFDM block, then the OFDM modulated signal can be represented by:

$$S_n(t) = \sum_{k=0}^{N-1} s_{n,k} e^{-j2\pi k\Delta f t}, \quad 0 \le t \le T_s \qquad (3)$$

where T_s, Δf , and N are the symbol duration , the subchannel space , and the number of sub-channels of OFDM respectively. For the receiver to demodulate the OFDM signal, the symbol duration should be long enough such that $T_s\,\Delta f = 1$ which is also called the orthogonal condition since it makes $e^{-j2\pi k\Delta f t}$ orthogonal to each other for different *k*. with the orthogonal condition the transmitted symbols $s_{n,k}$ can be detected at the receiver by (4)if there is no channel distortion.

$$s_{n,k} = \frac{1}{T_s}\int_0^{T_s} S_n(t) e^{-j2\pi k\Delta f t} dt \qquad (4)$$

The sampled version of the baseband OFDM signal $S_n(t)$ in (3) can be expressed as:

$$s_n\left(m\frac{T_s}{N}\right) = \sum_{k=0}^{N-1} S_{n,k} e^{-j2\pi k\Delta f m\frac{T_s}{N}} = \sum_{k=0}^{N-1} S_{n,k} e^{-j2\pi\frac{mk}{N}} \qquad (5)$$

which is the inverse discrete Fourier transform (IDFT) of the transmitted symbol $\{s_{n,k}\}_{k=0}^{N-1}$. It is straight forward to show that the demodulation can be performed using the DFT instead of integral in (4).

A cyclic prefix (CP) or guard interval is critical for OFDM to avoid inter-block interference (IBI) caused by the delay spread of wireless channels. They are usually inserted between adjacent OFDM blocks. If the length of the CP is equal or longer than the delay spread of the channel, ISI is completely eliminated by design. Without the CP, the length of the OFDM symbol is T_s, as shown in (3). With the CP, the transmitted signal is extended to $T = T_g + T_s$ and can be expressed as:

$$\tilde{s}_n(t) = \sum_{k=0}^{N-1} S_{n,k} e^{-j2\pi k\Delta f t}, \quad -T_g \le t \le T_s \tag{6}$$

It is obvious that $\tilde{s}_n(t) = s_n(t + T_s)$for $-T_g \le t \le 0$, which is why it is called the CP.

When we consider block by block processing over a linear time–invariant (within the block) frequency selective noisy channel the received symbols are given by:

$$r_n = H_n F^* s_n + z_n \tag{7}$$

where r_n is the received sequence and H_n is the $N\times N$ channel matrix is given by:

$$[H_n]_{i,j} = \begin{cases} h[n-L+i, i-j] & i \ge j \\ h[n-L+i, L+i-j-1] & i < j \end{cases} \tag{8}$$

F is the DFT matrix and F^* is the IDFT matrix, s_n is the data to be transmitted and z_n is white Gaussian noise in the time domain. After demodulation using DFT the received vector is given by

$$\hat{r}_n = F H_n F^* s_n + F z_n \tag{9}$$

In stationary conditions, H_n is circulant (because of cyclic prefix), and can be decoupled by the DFT matrix. As FHF^* is a diagonal matrix [7] we can equalize the received signal by simply adjusting the phase and amplitude of the received sequence[8].This property is the main advantage of DFT-OFDM because it simplifies the equalization process in multipath fading channel which otherwise would require a very complicated equalizers[9] in the single carrier transmission case. Most research in the field attempts to find better basis for OFDM that aim to imitate the same property as the DFT-OFDM [10-12]. However, this property is valid only in time–invariant frequency selective dispersive channel[13], when there is frequency offset in the receiver [14]or the channel is doubly selective (that is, time–frequency-selective)[4-6, 15].As this normally occurs in the rapidly fading wireless channel, this traditional methodology fails because the DFT is not able to diagonalize the channel matrix and Intercarrier interference (ICI) appears. In this situation the OFDM system needs complex equalizer [9, 16].

3. THE FrFT-OFDM System Model

a. The fractional Fourier transform and discrete implementation

The FrFT is the generalized formula for the Fourier transform that transforms a function into an intermediate domain between time and frequency. We may interpret it as a rotation operator in the time-frequency plane. This property makes the FrFT especially suitable for the processing of linear frequency modulation (LFM) also known as chirp-like signals or the signals passing through the linear time-varying system. The fractional Fourier transform of order a of an arbitrary function $x(t)$, with an angle α , is defined as[17]:

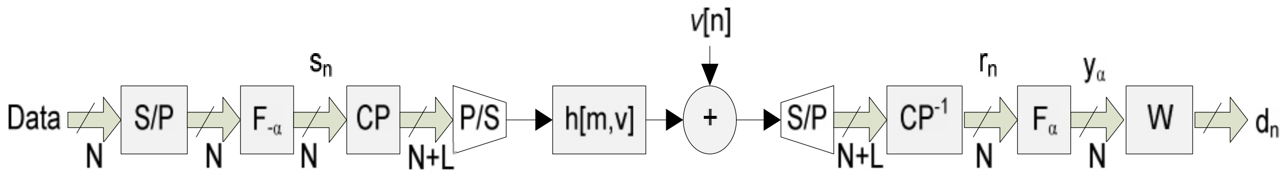

Fig.2 *The FrFT-OFDM system*

$$X_\alpha(u) = \int_{-N/2}^{N/2} x(t) K_\alpha(t,u)\, dt \tag{10}$$

where $K_\alpha(t,u)$ is the transformation Kernel and $\alpha = a\pi/2$ with $a \in \Re$.

$$K_\alpha(t,u) = \begin{cases} \sqrt{\dfrac{1 - j\cot\alpha}{2\pi}} \exp\left(j\dfrac{t^2+u^2}{2}\cot\alpha - j\,u\,t\csc\alpha\right) & \alpha \neq n\pi \\ \delta(t-u) & \alpha = n2\pi \\ \delta(t-u) & \alpha + \pi = n2\pi \end{cases} \tag{11}$$

The inverse FrFT (IFrFT) can be expressed as:

$$x(t) = \int_{-N/2}^{N/2} X_\alpha(u) K_{-\alpha}(t,u)\, du \tag{12}$$

The signal $x(t)$in (12)can be interpreted as decomposition to a basis formed by the orthonormal LFM functions in the u domain. The u domain is usually called the fractional Fourier domain, while the time and frequency domain can be considered as its special cases when $a = 0$ *and* 1 respectively. In engineering applications, there have been several discrete FrFT (DFrFT) algorithms with different accuracies and complexities. In our work, we select the DFrFT proposed in [17] to ensure that the transform kernel of the DFrFT and its inverse transform are orthogonal and reversible. The discrete FrFT formula of $X_\alpha[u]$is defined as:

$$X_\alpha[u] = \sum_{-N/2}^{N/2} x(t) K_\alpha[t,u] \tag{13}$$

Eq.(13) can also be written as matrix-vector multiplication, i.e.

$$X = F_\alpha \cdot x \tag{14}$$

where $X = [X_\alpha(0), X_\alpha(1) \ldots X_\alpha(N-1)]^T$.,

$\boldsymbol{x} = [x(0), x(1) \ldots x(N-1)]^T$ and $\boldsymbol{F}_\alpha$ is the $N \times N$ *FrFT* matrix where N is the number of samples and the FrFT-OFDM symbol length. Similarly the IDFrFT can be written as:

$$x = F_{-\alpha} \cdot X \tag{15}$$

where $F_{-\alpha} = F_\alpha{}^H$ and H is the complex conjugate transpose.

b. The FrFT-based OFDM system

The FrFT based OFDM system [1, 18,19] shown in Fig.2is similar to the conventional OFDM system with an FrFT modulator replacing the FFT modulator.

The subcarriers for the OFDM system are modulated by the Inverse Discrete FrFT (IDFrFT) where the transmitted data vector $d = [d_0\, d_1\, \ldots\ldots d_{N-1}]^T$ and the subcarriers vector $\boldsymbol{s} = [s_\alpha(0), s_\alpha(1) \ldots s_\alpha(N-1)]^T$are calculated from (15):

$$\boldsymbol{s} = F_{-\alpha} \,.\, d \tag{16}$$

At the receiver the subcarriers signals are demodulated using the Discrete FrFT (DFrFT) where the signal vector after demodulation (y_α) becomes:

$$y_\alpha = F_\alpha\, r = F_\alpha\, H\, s + F_\alpha\, n \tag{17}$$

$$y_\alpha = \widetilde{H}_\alpha d + \widetilde{n}_\alpha \tag{18}$$

where $r = H\,s + n$ is the received vector where n is the noise vector in the time domain and H is the channel matrix, $\widetilde{H}_\alpha = F_\alpha \overline{H} F_{-\alpha}$is the equivalent $N \times N$ channel matrix in the fractional domain and $\widetilde{n}_\alpha = F_\alpha n_\alpha$ is the noise vector in the fractional domain. $\widetilde{H}_\alpha$is a nondiagonal subcarrier channel matrix that introduces ICI, which is the case when the dispersive channel comprises a multipath doubly selective channel. This will make the symbol estimation task particularly complicated and complex equalizer is needed.

4. OFDM and FrFT-OFDM Equalization in Doubly Dispersive Channels

The linear maximum mean square error MMSE and zero-forcing (ZF) estimates [20] may be expressed as follows

$$\hat{d}_{MMSE} = \widetilde{H}_\alpha^H \left(\widetilde{H}_\alpha \widetilde{H}_\alpha^H + \gamma^{-1} I_N\right)^{-1} y_\alpha \tag{19}$$

$$\hat{d}_{ZF} = \widetilde{H}_\alpha^+ y_\alpha \tag{20}$$

where $\hat{d}_{MMSE}$ is the estimated data after MMSE equalization and $\hat{d}_{ZF}$is the estimated data after ZF equalization I_N is identity matrix with $N \times N$ elements and γ is the signal-to-noise ratio (SNR) and $\widetilde{H}_\alpha^+$ is the Moore-Penrose pseudo-inverse of the channel matrix in the Frequency Domain with $\alpha = \pi/2$ or the fractional domain with $0 < \alpha < \pi/2$. In

(19) and (20) perfect knowledge of the channel matrix *H* is assumed. Furthermore it is assumed that:

$$E\{d^{(i)}\} = E\{n^{(i)}\} = 0, \qquad (21)$$
$$E\{d^{(i)}d^{(i)H}\} = I, E\{d^{(i)}n^{(i)H}\} = 0, E\{n^{(i)}n^{(i)H}\} = \sigma^2 I$$

Various researches that aim to use less complex equalizers can be seen in [2, 20-23] .

5. Multimode Transmation Method Using OFDM and FrFT-OFDM

The proposed multimode transmission method adaptively changes some system components such as transmission rate or modulation, corresponding to the status of a transmitter, a receiver, data, or transmission environments. It facilitates an increase of transmission throughput and quality. Since the transmission environments change frequently and significantly in wireless communications, the multimode transmission is effective for wideband communications by means of the efficient use of resources such as a signal power and a spectrum bandwidth.

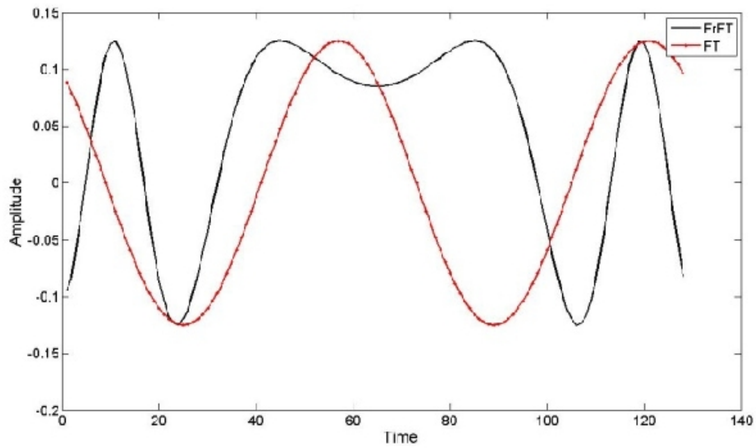

Fig.3 *FrFT-OFDM and OFDM Subcarriers*

As shown in Fig.3, FrFT-OFDM and OFDM have different subcarriers where OFDM subcarriers are orthogonal Frequencies with constant frequency per subcarrier and the FrFT-OFDM are orthogonal chirp signals with different frequencies per subcarrier. This difference appears as the difference of transmission performance in time invariant fading channels and time variant channels. That is, the degradation of transmission in time-variant fading channels environment can be suppressed by use of FrFT-OFDM, and that in multipath fading environments can be suppressed by use of OFDM. Therefore, if we construct a multimode system with FrFT-OFDM and OFDM, both advantages will be obtained. For example, in the case of indoor wireless transmission the radio waves suffer from time invariant multipath fading. In addition, deep time variant multipath fading occurs when the mobile terminal is moving especially with high speeds. In such case, the multimode transmission that switches FrFT-OFDM and OFDM corresponding to the transmission environments is effective, and the degradation could be suppressed. Fig.4shows the block diagram of multimode transmission with FRFT-OFDM and OFDM. In the transmitter, some modes including OFDM and different FrFT-OFDMs where each mode has a different α with different chirp signal are prepared, and data are transmitted in one mode corresponding to the transmission environment.

In the receiver, the transmitter mode is known and the data are decoded, the channel configuration can be determined through channel estimation and proper transmission mode must be selected and transmitted back to receiver. As a result, an increase in system performance will be expected in those severe environments.

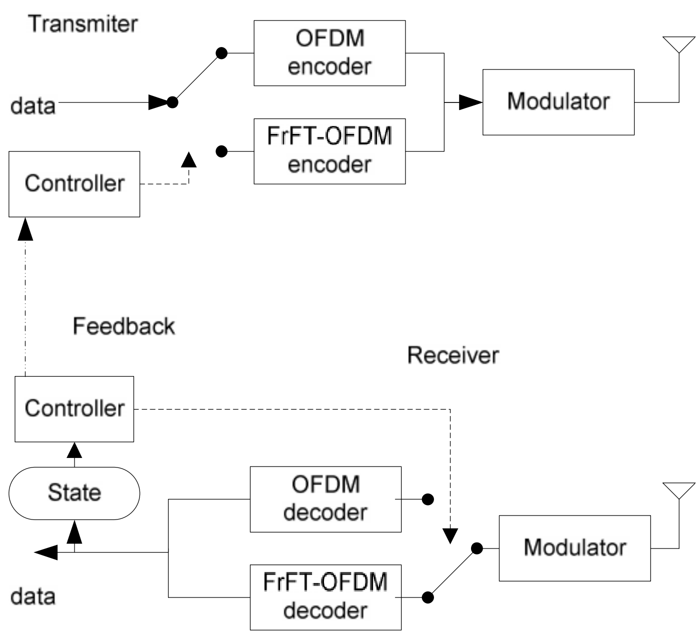

Fig.4 *FrFT-OFDM and OFDM Multimode System*

It is important to note that there is no need for separate FrFT-OFDM and OFDM blocks in both the transmitter and the receiver as shown in Fig.5. This is due to the fact that one of the properties of the fractional Fourier transform that it can have any fractional value $(0 \leq \alpha \leq \pi/2)$ where $\alpha = \pi/2$ is the Fourier transforms itself.

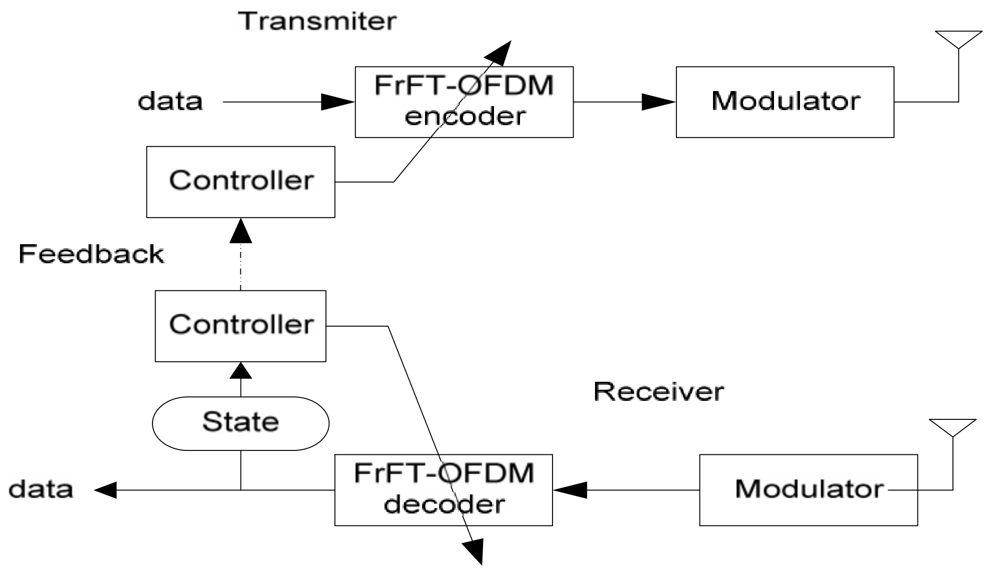

Fig.5*Multimode OFDM system with α selection*

6. Performance Analysis

The uncoded bit error rate (BER) performance of the traditional OFDM and the FrFT-OFDM systems are investigated in both channel environments:

a. Time invariant channel.
b. Time variant channel.

An OFDM system with N = 128, L = 8, and QPSK modulation is assumed. Rayleigh fading channels are simulated with an exponential power delay profile and root-mean-square delay spread of 3.The carrier frequency is $f_C = 10\ GHz$ and the subcarrier spacing is Δf = 20 kHz.

a. Time invariant channel:

In the time invariant channel environment the Doppler frequency $f_D =0$. The OFDM system uses the single tap equalizer and the FrFT-OFDM system use the MMSE equalizer. Fig.6 shows the BER performance for the both systems

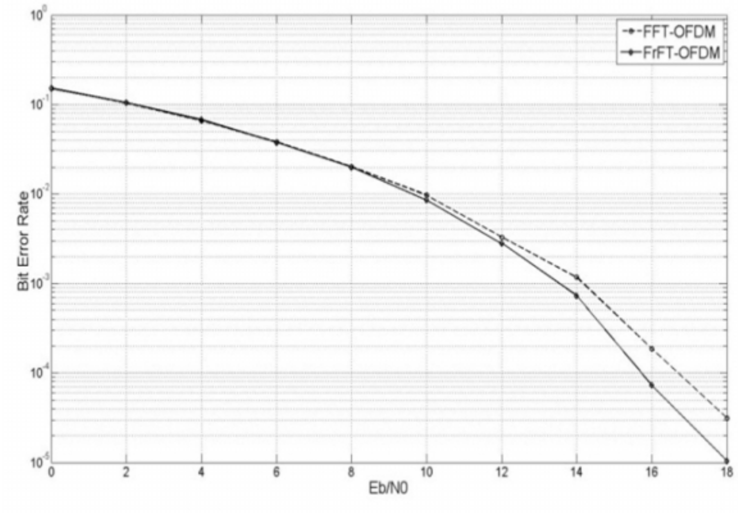

Fig.6 *OFDM and FrFT-OFDM BER comparison in time invariant channel environment*

From Fig.6 although the FrFT-OFDM system has superior performance, the OFDM system with the single tap equalizer has very good performance with less complexity. As a result it is better to use the OFDM mode in time invariant fading channel scenarios.

b. Time variant channel

In the time variant channel environment we consider the maximum Doppler frequency $f_D = 0.15\Delta f$. The MMSE equalizer was used for both The OFDM system and the FrFT-OFDM system. Fig.7 shows the BER performance for the both systems with the same block MMSE equalizer.

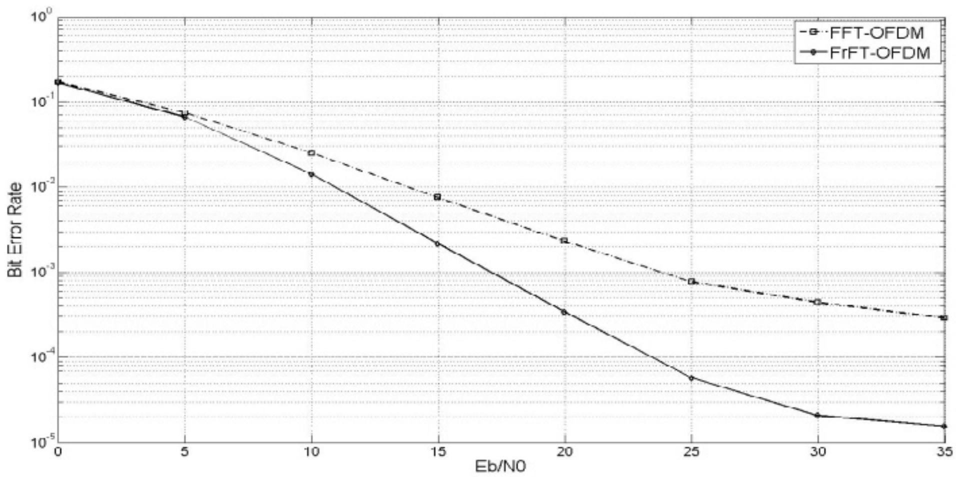

Fig.7 *OFDM and FrFT-OFDM BER comparison in time invariant channel environment*

From Fig.7 the FrFT-OFDM system has superior performance over the OFDM system with the same MMSE equalizer with the same complexity. As a result it is better to use the FrFT-OFDM mode in time variant fading channel scenarios.

7. Conclusion

In this paper a multimode transmission method using the FrFT-OFDM and OFDM is presented which can be applied to multiple transmission environments. Using the different transmission characteristics between FrFT-OFDM and OFDM in multipath fading and doubly selective multipath fading environments, good performance was obtained in both environments using the lowest complexity. The BERs of FrFT-OFDM and OFDM in fading environments were also calculated. The results demonstrated the relations between fading types and performances. Using the proposed multimode system will ensure the best transmission quality with the lowest complexity.

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