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# Comparison between Kalman Filter and PHD Filter in Multi-target Tracking

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## ABSTRACT

Tracking a maneuvering target weakens the performance of predictive-model-based Bayesian state estimators (Kalman Filter). Therefore, the Probability Hypothesis Density (PHD) filter was proposed to overcome this problem. In this paper, the performance of Kalman filter, modified Kalman filter, and PHD filter in tracking a highly maneuverable target is shown. All three algorithms to track a maneuverable target are applied. Monte Carlo simulation showed that the PHD filter provides promising performance compared to Kalman filter. In particular, the algorithm is capable of tracking multiple crossing maneuvering targets.

## KEY WORDS

Multi-target Tracking, Kalman Filter, Probability Hypothesis Density (PHD Filter)

## I INTRODUCTION

Tracking maneuvering targets is required in a wide range of civilian applications such as intelligent transportation system, air traffic control and surveillance. Therefore, researchers have concerned about this issue during the past several decades [1]. Surveillance systems are employing one or more sensors together with computer subsystems to interpret the environment. Typically sensor systems such as infrared (IR), sonar, and radar sensor. Reports measurement form diverse sources .the target tracking objective is to collect sensor data from field of view (FOV) containing one or more potential targets of interest and then partition sensor data into set of observation, or tracks that are produced by same object (or target),once tracks are formed and confirmed ,the number of target of interest can be estimated and quantities ,such as target velocity future predicted position and target classification characteristics ,can be computed from each track[2].

Since most surveillance systems must track multiple targets. multiple target tracking (MTT) is the most important tracking application fig. (1) [2]. shows the basic element of typically MTT system. Which have been formulated in the early papers by Wax [3] and Sittler [4] but these papers were written before the widespread application of the Kalman filtering techniques [5]. Bar-Shalom [6]and Singer[7,8] can be credited of modern MTT schemes that combine the data association techniques and Kalman filtering theory. Starting with Farina and Studer [9], a number of books, including [10-18], have been written to address the numerous problems involved in tracking multiple targets with one or more sensors [19] .

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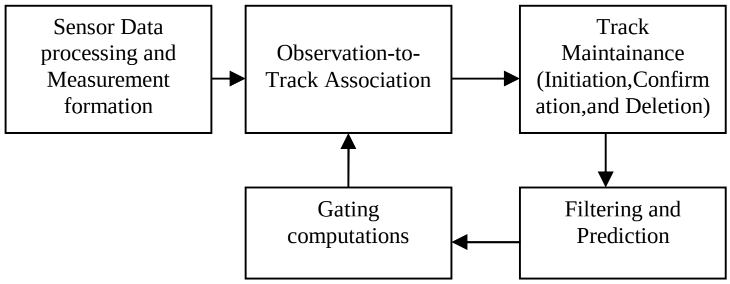


Fig.1. Basic elements of MTT system [2]

Gating, or measurement selection, is a necessary part of target tracking in clutter. The purpose of gating is to reduce computational expense by eliminating from consideration measurements which are far from the predicted measurement location. Gating is performed for each track at each scan by defining an area of surveillance space which is called the gate [20, 21]. All measurements positioned in the gate are selected and used for the track update while measurements not positioned in the gate are ignored for the purpose of the track update. The gate is usually formed in such a way that the probability of a target-originated measurement falling within the gate, provided that the target exists and is detected, is given by a gating probability PG which can be evaluated from the available track statistics. Since the size or volume of the gate is dependent on the tracking accuracy it therefore varies from scan to scan and from track to track, and the standard validation gate is ellipsoid [22]. Several classical data association methods exist. The simplest is probably the nearest neighbor (NN). In [20, 23], this is referred to as the nearest neighbor standard filter (NNSF) and uses only the closest observation to any given state to perform the measurement update step. The method can also be given as a global optimization, so the total observation to track statistical distance is minimized. Another multi target association method is Strongest Neighbor Filter (SNF)[24,25] It use the measurement with the strongest intensity (amplitude) in the neighborhood of the predicted target measurement location, known as the “strongest neighbor” measurement, as if it were the true one.

Another multi target tracking association method is the probability data association (PDA) [26], It estimates the states by a sum over all the association hypothesis weighted by the probabilities from the likelihood. an extension of it, is the joint probability data association (JPDA)[27,28] algorithm to multi targets. And the first developed by fortmann et al(1983)[29]. And another major approach is the multi hypothesis tracking (MHT)[2,30],and the first develop by Reid(1979)[31] which calculates every possible update hypothesis.

A new multi-target data association algorithm for radar tracking which we call the Fuzzy data association (FDA)[32]. This approach is formulated using the extended Kalman filter, and FDA is accomplished using the fuzzy logic algorithm. This technique is more robust and stable in heavy cluttered environment.

The measurements which correlate to a given track is processed by a filter to update the track parameter for these tracks that didn’t receive correlating observations, the previous predicted estimates are treated as the filtered estimates. Then, the predictions are made to the time when the next data scan is to be received [33]

As referred in [5] Kalman filter is used in prediction also their exist the probability hypothesis density (PHD)filtering approach, an attractive alternative to tracking unknown numbers of targets and their states in the presence of data association uncertainty, clutter, noise, and miss-detection. In particular, it has been discovered that the PHD filter has a closed form solution under linear Gaussian assumptions on the target dynamics and birth [34].

In section 2 we will present PHD filter and section 3 Kalman Filter and its modification in 4, section 5 the simulation results for maneuvering and non-maneuvering targets and the performance for each one

## II PHD filter

In several unpublished manuscripts written from 1993 to 1995, Michael Stein, C.L. Winter, and Robert Tenney introduced a multitarget tracking and evidential-accumulation concept called a "Probability Hypothesis Surface" (PHS) .A PHS is the graph of a probability distribution-the Probability Hypothesis Density (PHD)-that, when integrated over a region in target state space, gives the expected number of targets in that region. The PHD is uniquely defined by this property: Any other density function that satisfies it must be the PHD. In particular, the PHD is the expected value of the point process of a random track-set i.e. , of the density that, when integrated over a region in state space, gives the exact (random) number of targets in that region. In 1997 in the book Mathematics of Data Fusion was sketched the elements of a theoretical foundation for PHS/PHD. It was shown that the PHD is a first-order moment statistic of the random multitarget process and, consequently that from a computational perspective it is a multitarget analog of single-target constant-gain Kalman filters such as the α-β-γ filter.[35]

The probability hypothesis density (PHD) filtering approach, an attractive alternative to tracking unknown numbers of targets and their states in the presence of data association uncertainty, clutter, noise, and miss-detection.[34]

The PHD filter operates on the single-target state space and avoids the combinatorial problem that arises from data association. These salient features render the PHD filter extremely attractive. However, the PHD recursion involves multiple integrals that have no closed form solutions in general.[36]

The PHD represents the expectation, the integral of which in any region of the state space S is the expected number of objects in S.
The PHD is estimated instead of the multiple target posterior distribution as it is much less computationally expensive to do so. The time required for calculating joint multi-target likelihoods grows exponentially with the number of targets and is thus not very practical for sequential target estimation as this may need to be undertaken in real time.

The PHD is defined as the density, Dt|t (xt |Z1:t ), whose integral:

$$\int_{S} D_{t|t}(\chi_t \mid Z_{1:t})\mu(dx_t) = \int |X_t \cap S| \, f_{t|t}(X_t \mid Z_{1:t})\mu(dX_t) \qquad (1)$$

On any region S of the state space is the expected number of targets in S. The estimated object states can be detected as peaks of this distribution.

The derivation for the PHD equations is provided by Mahler [37], the prediction and update equations are given by:

$$D_{t|t-1}(\chi) = \gamma_t(\chi) + \int \phi_{t|t-1}(\chi, \zeta) D_{t-1|t-1}(\chi_{t-1})\mu(dx_{t-1}) \qquad (2)$$

$$D_{t|t}(\chi) = \left[ V(\chi) + \sum_{z \in Z} \frac{\Psi_{t,z}(\chi)}{K_t(z) + \langle D_{t|t-1}, \Psi_{t,z} \rangle} \right] D_{t|t-1}(\chi) \qquad (3)$$

Where

$$\phi_{t|t-1}(\chi, \zeta) = p_s(\zeta) \, f_{t|t-1}(\chi \mid \zeta) + b_{t|t-1}(\chi \mid \zeta) \qquad (4)$$

$$V(x) = 1 - P_D(x), \qquad (5)$$

$$k_t(z) = \lambda_t c_t(z) \qquad (6)$$

and

$$\Psi_{t,z} = P_D(\chi) g(z \mid x), \qquad (7)$$

In the prediction equation, $b_t$ is the PHD for spontaneous birth of a new target at time t, $P_S$ is the probability of target survival and $f_{t|t-1}(x_t|x_{t-1})$ is the single target motion distribution. In the data update equation, g is the single target likelihood function, $P_D$ is the probability of detection, $\lambda_t$ is the Poisson parameter specifying the expected number of false alarms and $c_t$ is the probability distribution over the state space of clutter points.[38]

## III Kalman filter

In 1960, R.E. Kalman published his famous paper describing a recursive solution to the discrete-data linear filtering problem [5]. Since that time, due in large part to advances in digital computing; the Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation.

The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) solution of the least-squares method. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown [39].The target can be modeled in discrete Markov form by

$$x(k+1) = Ax(k) + q(k) \qquad (8)$$

Where

x … n-dimensional target state vector,
A … state transition matrix, and
q(k) … zero mean white Gaussian noise with known covariance Q.

the M-dimensional measurement vector is modeled as

$$y(k) = Hx(k) - v(k) \quad (9)$$

Where

H … M×n measurement matrix, and

v(k) … zero mean white Gaussian measurement noise with covariance R.

Given the target dynamics and measurement model from (8) and (9) the kalman filter equations are driven in [40].which are given by

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)\left[y(k) - H\hat{x}(k/k-1)\right] \quad (10)$$

$$K(k) = P(k/k-1)H^T[HP(k/k-1)H^T + R \quad (11)$$

$$P(k/k) = [I - K(k)H]P(k/k-1) \quad (12)$$

$$\hat{x}(k+1/k) = A\hat{x}(k/k) \quad (13)$$

$$P(k+1/k) = AP(k/k)A^T + Q \quad (14)$$

Where

P(k/k) … the estimated covariance matrix at scan k, and

P(K+1/k) … the predicted covariance matrix at scan k+1 given scan k.

## IV Modified Kalman filter

We assume a threshold to make reset to error covariance matrix P to its initial value P0.

Table 1. Pseudo-Code for Modified KalmanFilter

```
Given Z, H ,P0,X0, and TH  % TH  is the threshold %%

Res=(Z-H*X0);
   if (norm(Res)>=TH)
     P0=P01;
  end
Xest=X0+K*Res;       % Filtering State %%
Pest=(eye(4)-K*H)*P0; % Filtering Error Covariance %%
```

## V Simulation Results

We apply three different algorithms which are Kalman filter, modified with reset error, and PHD filter to five scenarios first one is linear motion (bomber) and three different scenarios for fighter and the last for trainers.

### 5.1 Bomber

In this section we will show bomber air craft in linear motion and discuss the three output of the filters used.

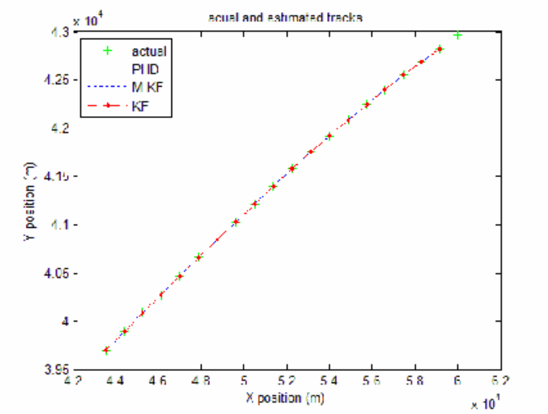

Fig.2. Actual trajectory for bomber

In this scenario, 19 radar scan are done on a part on land (4.3, 6.2)*$10^4$ (m) and using the three filters, we see that for linear motion the curves for all filters and the error criteria are nearly to be identical.

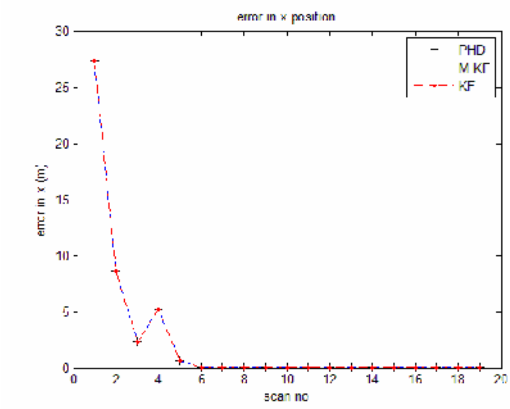

Fig.3. Error in X position

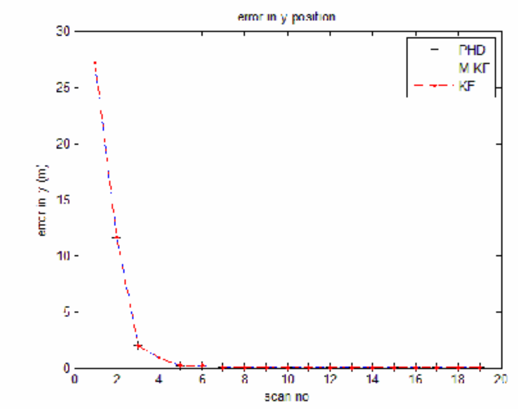

Fig.4. Error in Y position

Table 2. Error analysis for bomber

| Kind of filter | Error in X position (m) | Error in Y position (m) | Error criteria |
|---|---|---|---|
| Kalman filter | 2.974641 | 1.055708 | 3.156424 |
| Modified Kalman | 2.97703 | 1.056524 | 3.158947 |
| PHD filter | 2.97703 | 1.056524 | 3.158947 |

**5.2 Fighter 1**

In this section we will show the difference between the 3 algorithms during maneuver motion by fighter air craft

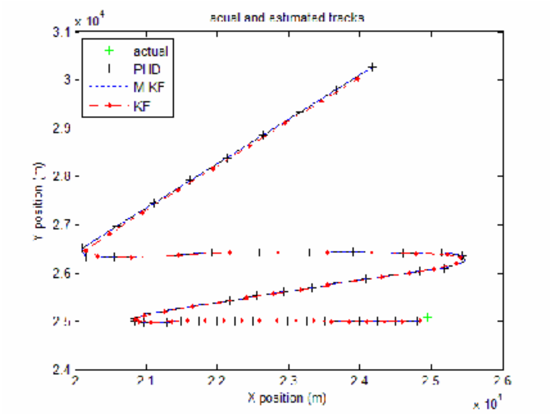


Fig.5. Actual trajectory for fighter1

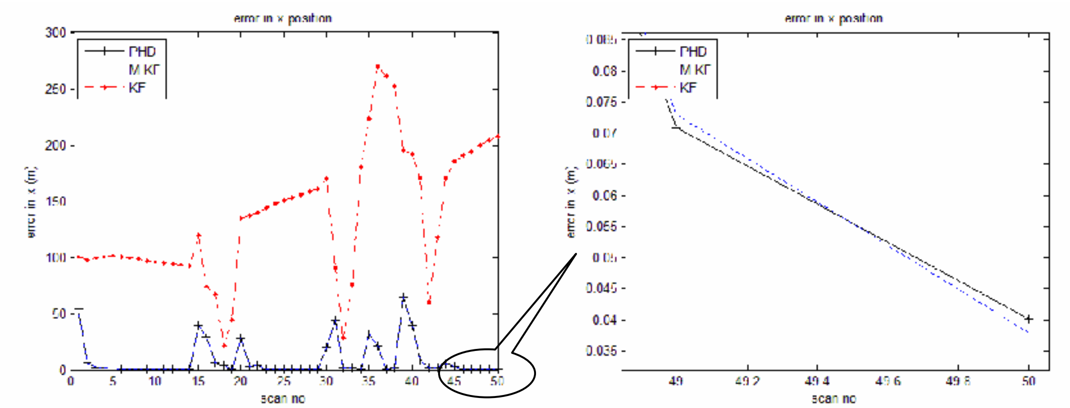


Fig.6. Error in X position

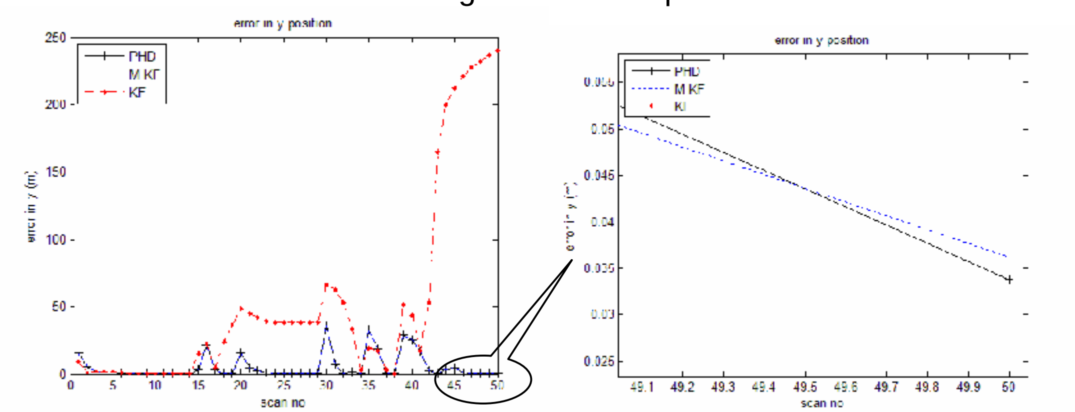


Fig.7. Error in Y position

In this scenario, 50 radar scan are done on a part on land (3.1, 2.6)*$10^4$ (m) and using the three filters, we see that for maneuver motion the curves of the filters and the error criteria are different and Kalman filter has large error 147.2178 and the difference between the two other filters is 0.000072

Table 3. Error analysis for fighter 1

| Kind of filter | Error in X position (m) | Error in Y position (m) | Error criteria |
|---|---|---|---|
| Kalman filter | 136.8142 | 54.35944 | 147.2178 |
| Modified Kalman | 4.180211 | 6.677924 | 7.878378 |
| PHD filter | 4.179974 | 6.677986 | 7.878304 |

### 5.3 Fighter 2

Another scenario for maneuver motion will be shown in this section also by fighter air craft

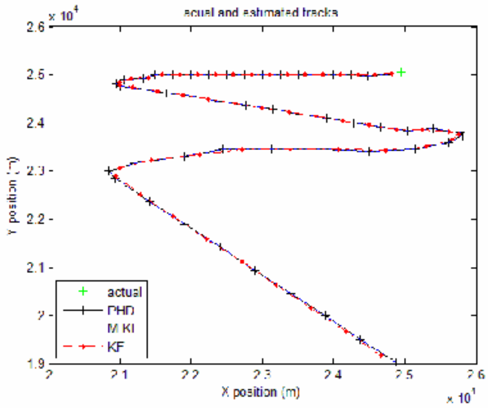


Fig.8. Actual trajectory for fighter 2

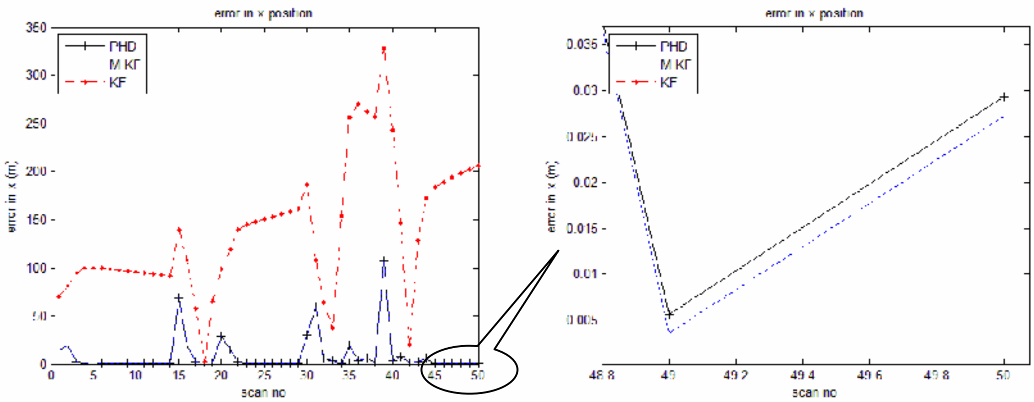


Fig.9. Error in X position

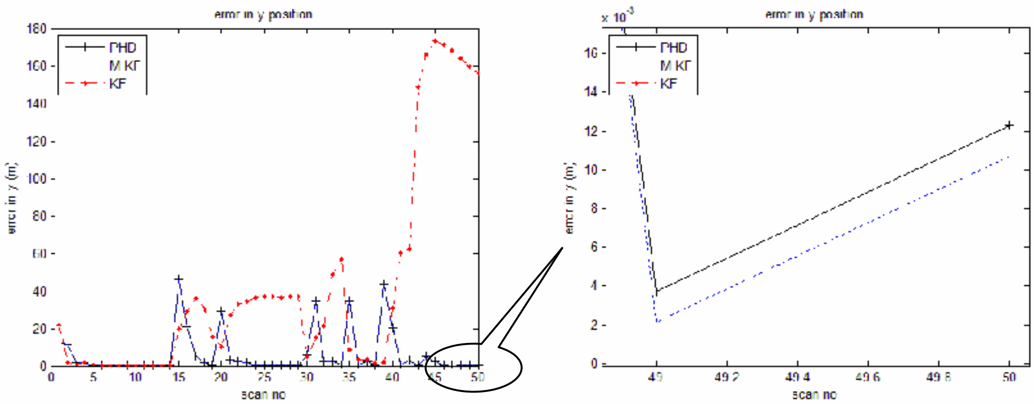


Fig.10. Error in Y position

In this scenario, 50 radar scan are done on a part on land (2.6, 2.6)*$10^4$ (m) and using the three filters, we see that for maneuver motion the curves of the filters and the error criteria are different and Kalman filter has large error 141.5815 and the difference between the two other filters is 0.00011

Table 4. Error analysis for fighter 2

| Kind of filter | Error in X position (m) | Error in Y position (m) | Error criteria |
|---|---|---|---|
| Kalman filter | 133.9246 | 45.92965 | 141.5815 |
| Modified Kalman | 7.709743 | 6.435743 | 10.04285 |
| PHD filter | 7.709715 | 6.435936 | 10.04296 |

### 5.4 Fighter 3

Also this section will see another way for maneuver motion by fighter air craft

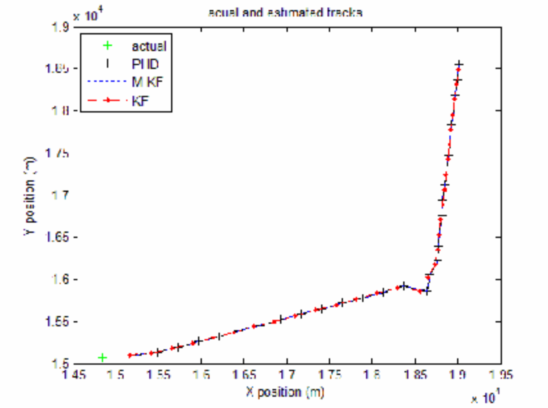


Fig.11. Actual trajectory for fighter 3

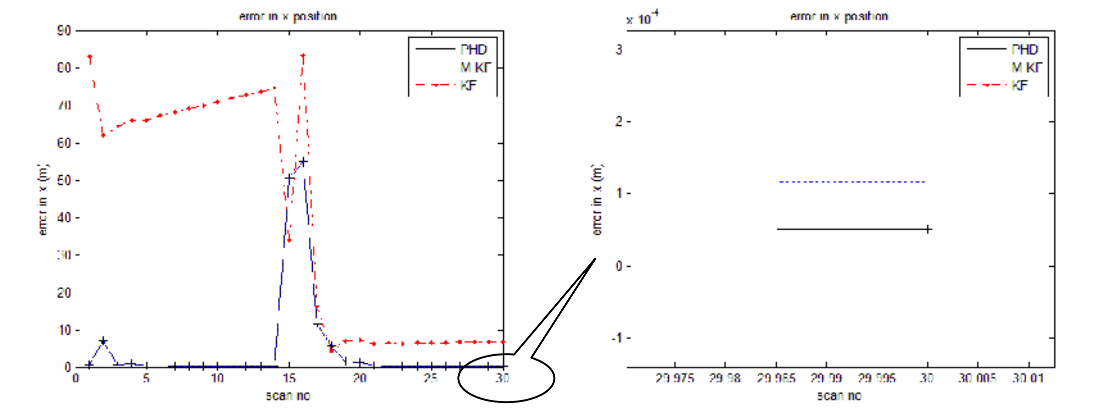


Fig.12. Error in X position

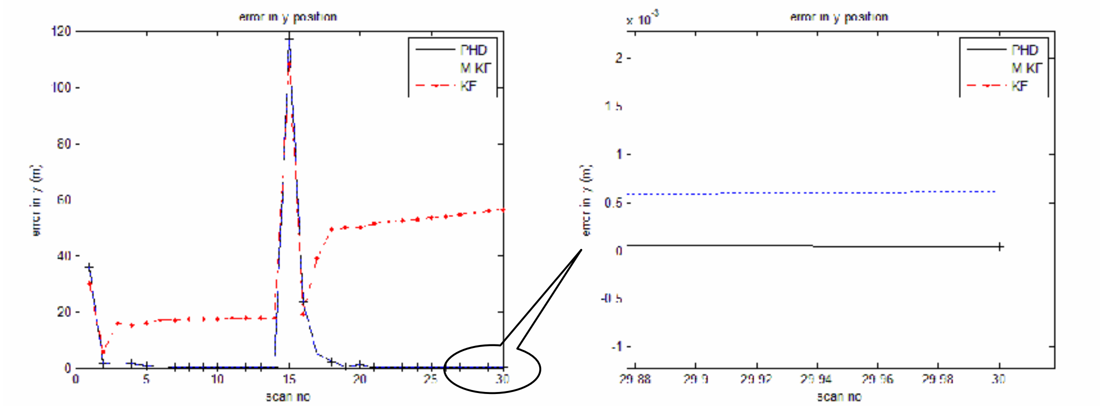


Fig.13. Error in Y position

In this scenario, 30 radar scan are done on a part on land (1,95, 1,9)*$10^4$ (m) and using the three filters, we see that for maneuver motion the curves of the filters and the error criteria are different and Kalman filter has large error 56.38372 and the difference between the two other filters is 0.000138

Table 5. Error analysis for fighter 3

| Kind of filter | Error in X position (m) | Error in Y position (m) | Error criteria |
|---|---|---|---|
| Kalman filter | 44.03945 | 35.20868 | 56.38372 |
| Modified Kalman | 6.013361 | 4.159153 | 7.311571 |
| PHD filter | 6.013213 | 4.159125 | 7.311433 |

**5.5 Trainer**

Another maneuver by different type of air craft called trainer

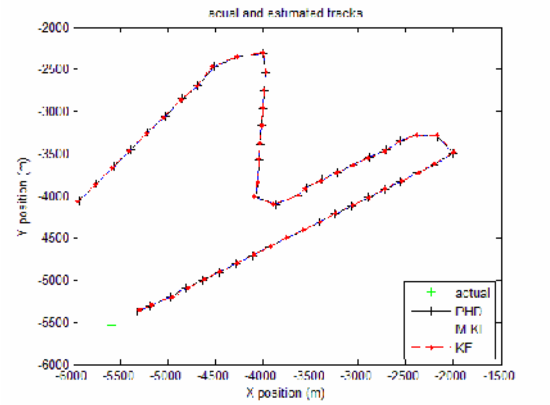


Fig.14. Actual trajectory for trainer

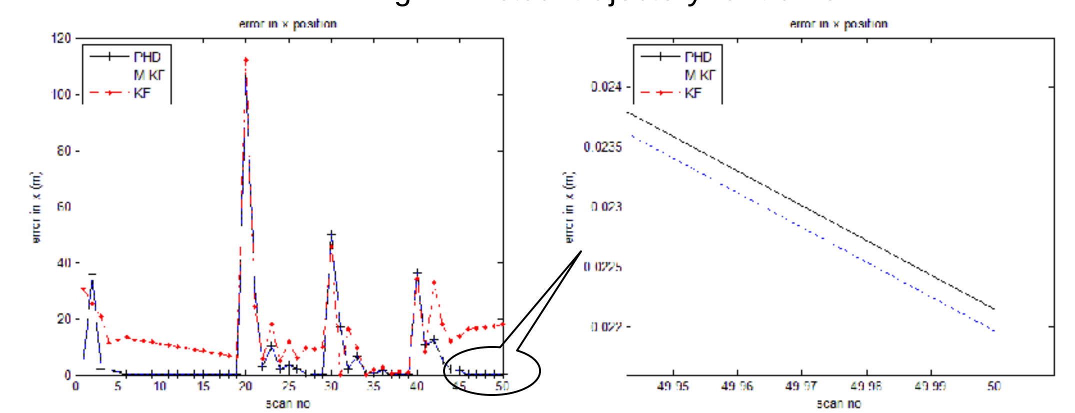


Fig.15. Error in X position

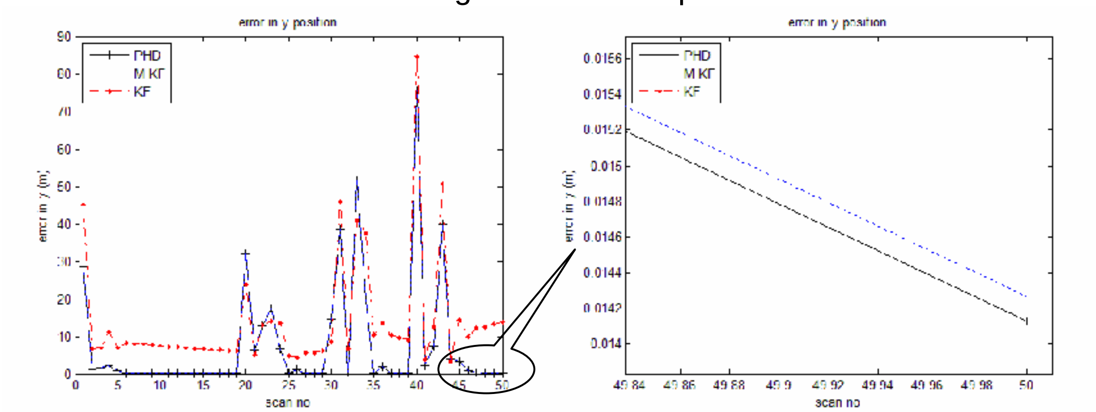


Fig.16. Error in Y position

In this scenario, 50 radar scan are done on a part on land (4.5, 4)*$10^4$ (m) and using the three filters, we see that for maneuver motion the curves of the filters and the error criteria are different and Kalman filter has large error 20.74106 and the difference between the two other filters is zero

Table 6. Error analysis for trainer

| Kind of filter | Error in X position (m) | Error in Y position (m) | Error criteria |
|---|---|---|---|
| Kalman filter | 16.09614 | 13.08075 | 20.74106 |
| Modified Kalman | 7.851098 | 7.46157 | 10.83119 |
| PHD filter | 7.851084 | 7.461582 | 10.83119 |

## VI CONCLUSION

We showed that modified Kalman Filter provides promising performance which is nearly typically the performance of the PHD Filter. This code is written by Matlab which is easy to change it to any other language as c++ or lab view to be used in real time application. In particular, the algorithm is capable of tracking multiple maneuvering targets that cross each other and the number of target tracked known from the number of stages that in the algorithm.

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