A novel multisensor integration approach for distributed detection systems with data fusion

By

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Abstract:

In this paper, we consider a binary distributed detection system in which a system of multiple sensors monitors a common volume and provides relevant binary decisions about the state of the environment to a data fusion center. The fusion center combines the binary decisions of the individual distributed sensors into a final global decision. We propose a new hard decision integration method for multiple sensor decision fusion systems. The proposed hard decision fusion method determines the false alarm probabilities and the detection probabilities that yield maximum performance. The performance of the proposed method is provided in case of Rayleigh distributed observations and is proved to be simple and efficient.

Keywords:

Binary integration, distributed detection, decision fusion, multisensor detection

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1. Introduction:

Interest in signal processing with multiple-sensor systems has surfaced with anticipated applications in target detection using geographically distributed sensors [1]. We consider the problem of decision fusion in a multiple sensor system based on a system of multiple distributed sensors and a fusion center. This is considered as a binary hypothesis testing problem with two hypotheses; \( H_0 \) designating signal absent and \( H_1 \) designating signal present. The distributed sensors monitor the same object scene and transmit their information (binary local decisions) about a hypothesis to the fusion center. The fusion center is responsible for combining the individual sensors decisions and making a final global decision about the same hypothesis. Such systems are expected to increase the reliability of the detection and be immune to noise interference and failure [2-4].

The Neyman-Pearson fusion of statistically independent sensor decisions has been investigated in many previous literatures [4-6]. However, most of them have concentrated on optimizing system performance at the individual sensors or at the fusion center but not both. The global optimum solution of a distributed decision system consists of a set of strongly coupled conditions. Thus explicit solutions of sensor thresholds and fusion rule are not obtained. Instead, the solution of a distributed sensor system is usually obtained by adopting various Boolean algebraic combinations as fusion strategies [6, 7]. Since the Boolean algebraic fusion rules are optimum only in case of identical sensors, the use of such fusion rules does not obtain the optimum solutions in case of non identical sensors (practical case).

In this paper, a new hard decision integration method for multiple sensor distributed detection systems, in terms of both the sensors and the fusion center, according to Neyman-Pearson strategy, is proposed. The proposed algorithm determines, for a given global false alarm probability, the corresponding optimum setting of the thresholds both at the fusion center and at the sensors. It scans all possible solutions and picks the solution that gives the largest global detection probability. The proposed method is found to be simple, accurate and fast. It is worth noting that the execution time of the proposed algorithm is mainly dependent on the number of considered sensors no matter they are identical or not, although from an analytic point of view finding optimum operating point of \( n \) sensors is far more difficult than finding a common optimum operating point in case of identical sensors. The performance analysis of the proposed algorithm is provided in case of Rayleigh distributed observations. The remainder of this paper is organized as follow. A brief review of the distributed sensor decision fusion systems and the problem formulation are presented in Section 2. The solution to
the distributed sensor decision fusion systems according to the Neyman-Pearson criterion, in terms of the sensors thresholds and the data fusion rule, is proposed in Section 3. The performance of the proposed algorithm and several optimization examples are reported in Section 4. Finally, concluding remarks are given in Section 5.

2. Problem Statement:

Given n-sensor distributed decision fusion system with data fusion, where each sensor observes a common volume and receives observations from observed targets. Each local sensor decide whether there is a target or not, according to a comparison of its likelihood ratio with a sensor threshold. If the sensor's likelihood ratio greater than the sensor's threshold, its local decision will be one \( u_i = 1 \). If the sensor's likelihood ratio lower than the sensor's threshold, its local decision will be zero \( u_i = 0 \), i.e. the local sensor decisions are either ones or zeros \( u_i = 0, 1, i = 1, 2, \ldots, n \). The local sensors send their local decisions to a data fusion center. The data fusion center combines the sensors local decisions \( \{ u_i \}'s \) to derive a global decision \( u_0 \). The objective is to optimize the structures of both the sensors and the fusion center according to Neyman-Pearson criterion. This is a distributed hypothesis testing problem with two hypotheses \( H_0 \) and \( H_1 \). The Neyman Pearson (NP) optimization strategy maximizes the global detection probability (GDP) for a desired global false alarm probability (GFAP). The prior knowledge needed for applying the NP strategy comprises the desired GFAP, and the probability density functions of the sensor observations conditioned to hypotheses \( H_0 \) and \( H_1 \); i.e. \( P(y_k | H_0) \) and \( P(y_k | H_1) \), \( k = 1, 2, \ldots, n \).

Let \( u \) be the vector formed of the decisions \( \{ u_1, u_2, \ldots, u_n \} \). Thus \( u \) belongs to the \( n \)-dimensional binary space \( Y^n \). The global missed-detection and false alarm probabilities are respectively the probabilities \( \Pr(u_0 = 0|H_i) \) and \( \Pr(u_0 = 1|H_0) \). According to the NP criterion, it is desired to determine the fusion strategy \( \Pr(u_0 = 1|u) \) and the sensor decision strategies \( \Pr(u_k = 1|y_k), 1 \leq k \leq n \), which minimizes \( \Pr(u_0 = 0|H_1) \) under the constraint

\[
\Pr(u_0 = 1|H_0) \leq GFAP
\]  

(1)

The optimal fusion strategy is deterministic \([9, 10]\) and is given as

\[
L(u) = \frac{\Pr(u|H_1)}{\Pr(u|H_0)},
\]  

(2)
The optimal decision strategy for the $k^{th}$ sensor is [4]
\[
\text{pr}(u_k = 1 | y_k) = \begin{cases} 
1 & \text{if } P(y_k | H_1) C_{k1} - \lambda^* P(y_k | H_0) C_{k0} > 0 \\
0 & \text{if } P(y_k | H_1) C_{k1} - \lambda^* P(y_k | H_0) C_{k0} \leq 0,
\end{cases}
\]
(3)
or equivalently,
\[
L(y_k) C_{k1} > \lambda^* C_{k0}, \quad 1 \leq k \leq n,
\]
(4)
where
\[
L(y_k) = \frac{P(y_k | H_1)}{P(y_k | H_0)}, \quad 1 \leq k \leq n.
\]
(5)

It is clear from (4) and (5) that the $k^{th}$ sensor decision strategy, $1 \leq k \leq n$, is deterministic and it partitions the observation space $Y$ into two disjoint subspaces, $Y_k$ and $\hat{Y}_k$ such that
\[
Y_k = \{ y_k : L(y_k) C_{k1} > \lambda^* C_{k0} \}, \quad 1 \leq k \leq n.
\]
(6)

In [4, 9], it is shown that the fusion strategy (3) is equivalent to
\[
V(u) = \sum_{k=1}^{n} a_k u_k \left( \sum_{k=0}^{n-1} t_0 \right),
\]
(7)
where
\[
a_k = \ln \left[ \frac{p_{d_k} (1 - p_{f_k})}{p_{f_k} (1 - p_{d_k})} \right], \quad 1 \leq k \leq n,
\]
(8)
\[
t_0 = \ln \lambda^* - \sum_{k=1}^{n} \ln \left( \frac{1 - p_{d_k}}{1 - p_{f_k}} \right).
\]
(9)

### 3. Proposed integration approach:

The proposed method determines for a given GFAR, the corresponding optimum setting of the fusion center threshold ($t_0$) and of the operating points of the sensors ($p_{f_k}, p_{d_k}$), $1 \leq k \leq n$, as well as the achieved GDP. In the following a parameter $w$ is
introduced to scan the permissible values of the fusion center threshold.

The proposed algorithm has the following steps:

1. For every sensor determine the ROC corresponding to a likelihood ratio decision rule either explicitly, \( pd_k = \phi_k(pf_k) \), or parametrically \( pd_k = \beta_k(\lambda_k) \), \( pf_k = \alpha_k(\lambda_k) \), with the threshold \( \lambda_k \) as a parameter, \( 1 \leq k \leq n \).

2. Assign \( w \) its smallest possible value \( \delta \).

3. Select arbitrary operating points \( (pf_k, pd_k) \) on the sensors ROCs such that
   \[
   \sum_{k=1}^{n} pf_k \leq GFAP
   \]  
   (10)

4. Associate with every vector \( u \) of \( Y^* \) the corresponding value of \( \Pr(u|H_0) \) and \( \Pr(u|H_1) \).

5. Rearrange the values of \( \Pr(u|H_0) \) and \( \Pr(u|H_1) \) in a descending order.

6. Find the minimum value \( v_\lambda \) such that
   \[
   \Pr[V(u)>v_\lambda|H_0] \leq GFAP,
   \]  
   (11)

7. Evaluate the probability \( \hat{GDP} \) as
   \[
   \hat{GDP} = \sum_{u \in D} \Pr(u|H_1).
   \]  
   (12)

8. Calculate the threshold \( t_0 \) as
   \[
   t_0 = w v_\lambda + (1-w) v_{\lambda-1}, \quad 0 < w \leq 1.
   \]  
   (13)

9. For every sensor calculate the coefficients \( C_{i,k}, i = 0,1, k = 1,2,\ldots,n \), from (4). Update the operating points \( (pf_k, pd_k) \) according to the new thresholds in the set of decision rules (5).

10. For the assigned value of \( w \) repeat the steps from (4) to (9) often enough until the difference between the estimated values of \( GDP \) in successive iterations becomes zero. Skip the value of \( w \) if this condition is not satisfied.

11. Increment \( w \) and repeat the steps from (3) to (11) till all the possible values of \( w \) are exhausted.
Scan the recorded values of $w$ and their associated steady state estimates $\hat{G}\hat{D}P$. The value of $w$ which results in the largest value of $\hat{G}\hat{D}P$ is the optimum choice ($w_{\text{opt}}, \hat{G}\hat{D}P_{\text{max}}$).

4. Performance of the proposed integration approach:

We consider the case of Rayleigh distributed observations. The probability distributions under both hypotheses are given by [9]

$$P(y_k | H_0) = \exp(-y_k),$$  \hspace{1cm} (14)

$$P(y_k | H_1) = d_k \exp(-d_k y_k), \quad d_k > 0, k = 1,2,\ldots,n, \text{ for } y_k \geq 0,$$  \hspace{1cm} (15)

$$P(y_k | H_1) = 0, k = 1,2,\ldots,n, i = 0,1.$$  \hspace{1cm} (16)

The coefficient $d_k, k = 1,2,\ldots,n$, is related to the signal to noise ratio at each individual sensor ($r_k$) as

$$d_k = \frac{1}{1 + r_k}, \quad k = 1,2,\ldots,n.$$  \hspace{1cm} (17)

The detection and false alarm probabilities are related, in terms of the coefficient $d$ as:

$$pd_k = pf_k^{(1/(1+r_k))} = pf_k^{d_k}, \quad k = 1,2,\ldots,n.$$  \hspace{1cm} (18)

We consider three cases. The first case considers a distributed decision fusion system with two non-identical sensors. In this case, the global optimization is obtained as described in [7, 10]. The result is plotted in Fig. 1 in case of $d_1 = 0.1$ and $d_2 = 0.4$. Figure 1 shows the OR and the AND fusion rules (the optimal ROCs in case of two sensors [4, 9, 30]). The AND fusion rule is optimum at very low false alarm probabilities while the OR fusion rule is optimum at high false alarm probabilities. The ROC achieved by the proposed algorithm is shown in Fig. 2. From Figures 1 and 2 it is readily seen that the ROC achieved by the proposed algorithm is indeed the convex hull of the ROCs obtained with the OR and AND fusion rules; i.e. it is the optimal ROC.

The second case is the case of identical sensor where the optimum fusion rule reduces to $K$-out-of-$n$ fusion rule. For a specified $K$, we solve the equation
GFAP = \sum_{i=K}^{n} c_i^n pf^i (1 - pf)^{n-i},

(19)

for \( pf \) and then determine the corresponding \( pd \) from the common ROC of the identical sensors. The global detection probability, for the specified \( K \), is calculated from

\[
GDP = \sum_{i=k}^{n} c_i^n pd^i (1 - pd)^{n-i}.
\]

(20)

For every desired value of GFAP, there is an optimum integer value of \( K \) that maximizes the GDP.

The second case considers a distributed decision fusion system with four identical sensors in case of Rayleigh distributed observations with \( d = 0.02 \). The ROCs corresponding to the \( K \)-out-of-\( n \) fusion rules, \( K = 1, 2, 3, 4 \), and to the proposed Neyman Pearson fusion of the decisions of the four identical sensors are given in Table 1. The points in the last row of Table 1 are determined using the proposed algorithm. It can easily be verified that the last ROC is the convex hull of the other ROCs. The third case considers the solution of six non-identical sensors in case of Rayleigh distributed observations at different small values of GFAPs. Table 2 shows the results. It is shown that the proposed integration method is very efficient. The proposed algorithm can be used to optimize distributed decision fusion systems according to criteria other than the Neyman Pearson criterion. For example, in multiple sensor distributed IFF (Identification Friend or Foe) systems two types of decision errors may be committed: identifying a hostile target as a friendly one, or conversely identifying a friendly target as a hostile one. In heavy air combat situations both types of errors are almost equally significant. Hence, it is more convenient to optimize the decision strategy of the fusion center according to the Ideal observer criterion rather than the Neyman Pearson criterion.
Figure (1): Comparison of ROCs of two different sensors using "AND" and "OR" fusion rules

Figure (2): The global ROC of two different sensors using the proposed approach
Table (1): Comparison of ROC's of four-sensor distributed detection system with $K$-out-of-$n$ fusion rules with the proposed integration method ($d = 0.02$)

<table>
<thead>
<tr>
<th>GFAP</th>
<th>GDP ($K = 1$)</th>
<th>GDP ($K = 2$)</th>
<th>GDP ($K = 3$)</th>
<th>GDP ($K = 4$)</th>
<th>$W_{opt}$</th>
<th>$GDP_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00015</td>
<td>.99889</td>
<td>.9962</td>
<td>.9764</td>
<td>.8318</td>
<td>.56358</td>
<td>.99889</td>
</tr>
<tr>
<td>.00025</td>
<td>.99905</td>
<td>.9967</td>
<td>.9786</td>
<td>.8471</td>
<td>.56898</td>
<td>.99905</td>
</tr>
<tr>
<td>.00035</td>
<td>.99926</td>
<td>.9970</td>
<td>.9800</td>
<td>.8529</td>
<td>.57213</td>
<td>.99926</td>
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<tr>
<td>.00045</td>
<td>.99928</td>
<td>.9972</td>
<td>.9810</td>
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<td>.57951</td>
<td>.99928</td>
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<tr>
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<td>.9974</td>
<td>.9818</td>
<td>.8621</td>
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<td>.99932</td>
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<td>.00065</td>
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<td>.99780</td>
<td>.9834</td>
<td>.8691</td>
<td>.61005</td>
<td>.99941</td>
</tr>
</tbody>
</table>
**Table (2):** Solution of a six non identical sensors distributed detection system at different values of GFAP's in case of Rayleigh distributed observations

For $GFAP = 10^{-4}$, $w_{opt} = 0.5892341$

<table>
<thead>
<tr>
<th>$d_k$</th>
<th>$t_k$</th>
<th>$pf_k$</th>
<th>$pd_k$</th>
<th>$a_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>8.4145</td>
<td>2.216384$x10^{-4}$</td>
<td>0.776908</td>
<td>9.66198</td>
</tr>
<tr>
<td>0.04</td>
<td>8.4657</td>
<td>2.105587$x10^{-4}$</td>
<td>0.712746</td>
<td>9.37430</td>
</tr>
<tr>
<td>0.05</td>
<td>8.5183</td>
<td>1.997695$x10^{-4}$</td>
<td>0.653170</td>
<td>9.15115</td>
</tr>
<tr>
<td>0.06</td>
<td>8.5723</td>
<td>1.892743$x10^{-4}$</td>
<td>0.597896</td>
<td>8.96883</td>
</tr>
<tr>
<td>0.07</td>
<td>8.6277</td>
<td>1.790768$x10^{-4}$</td>
<td>0.546654</td>
<td>8.81468</td>
</tr>
<tr>
<td>0.08</td>
<td>8.7234</td>
<td>1.626792$x10^{-4}$</td>
<td>0.486678</td>
<td>8.72063</td>
</tr>
</tbody>
</table>

$t_0 = 5.23567, \ G\hat{D}P_{max} = 0.966789$

For $GFAP = 10^{-5}$, $w_{opt} = 0.7178023$

<table>
<thead>
<tr>
<th>$d_k$</th>
<th>$t_k$</th>
<th>$pf_k$</th>
<th>$pd_k$</th>
<th>$a_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>15.2456</td>
<td>2.392884$x10^{-7}$</td>
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<td>0.04</td>
<td>15.3365</td>
<td>2.184913$x10^{-7}$</td>
<td>0.541474</td>
<td>15.50280</td>
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<tr>
<td>0.05</td>
<td>15.4307</td>
<td>1.988468$x10^{-7}$</td>
<td>0.462302</td>
<td>15.27965</td>
</tr>
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<td>0.06</td>
<td>15.5283</td>
<td>1.803530$x10^{-7}$</td>
<td>0.393883</td>
<td>15.09733</td>
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<td>0.07</td>
<td>15.6295</td>
<td>1.630027$x10^{-7}$</td>
<td>0.334853</td>
<td>14.94318</td>
</tr>
<tr>
<td>0.08</td>
<td>15.7301</td>
<td>1.231890$x10^{-7}$</td>
<td>0.261028</td>
<td>14.71891</td>
</tr>
</tbody>
</table>

$t_0 = 9.1046, \ G\hat{D}P_{max} = 0.9310265$

For $GFAP = 10^{-6}$, $w_{opt} = 0.8290315$

<table>
<thead>
<tr>
<th>$d_k$</th>
<th>$t_k$</th>
<th>$pf_k$</th>
<th>$pd_k$</th>
<th>$a_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>22.0735</td>
<td>2.591706$x10^{-10}$</td>
<td>0.515712</td>
<td>22.13640</td>
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<td>0.04</td>
<td>22.2071</td>
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<td>0.05</td>
<td>22.3467</td>
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<td>0.06</td>
<td>22.4926</td>
<td>1.704503$x10^{-10}$</td>
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<td>21.44326</td>
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<tr>
<td>0.07</td>
<td>22.6449</td>
<td>1.463613$x10^{-10}$</td>
<td>0.204918</td>
<td>21.28911</td>
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<td>0.08</td>
<td>22.8219</td>
<td>1.183045$x10^{-10}$</td>
<td>0.157830</td>
<td>21.09367</td>
</tr>
</tbody>
</table>

$t_0 = 14.1547, \ G\hat{D}P_{max} = 0.9048920$
6. Conclusions:

In this paper, a new hard decision integration method for multiple sensor distributed detection systems has been proposed. The proposed algorithm determines, for a given false alarm probability, the corresponding optimum setting of the thresholds both at the fusion center and at the sensors. It scans all possible solutions and picks the solution that gives the largest global detection probability. Examples of global optimization of several distributed decision fusion systems have been presented. The proposed algorithm is found to be simple, accurate and fast. It is worth noting that the execution time of the proposed algorithm is mainly dependent on the number of considered sensors no matter they are identical or not. It is also shown that the proposed algorithm can be used to optimize distributed decision fusion systems according to criteria other than the Neyman-Pearson one.

References:

