OPTIMIZING THE DESIGN STRATEGY OF INTERCEPT RECEIVERS FOR FREQUENCY HOPPING CW SIGNALS

By

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Abstract:

In this paper an optimal strategy is proposed to design intercept receivers that can measure and track the frequencies of simultaneous CW frequency hopping signals with a certain resolution at a 100% intercept probability. The strategy goal is to minimize the frequency measurement time and maximize the discernible frequency-hopping rate.

Keywords:

Intercept receiver, frequency measurement, frequency hopping, group delay

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1. INTRODUCTION

An intercept receiver receives and measures simultaneous unknown signals. A set of parameters is used to evaluate its performance [1]. However, improving one parameter may influence another one. Therefore, the design must consider the overall capability of the receiver and make design tradeoffs among the parameters [2]. This paper studies the maximization of frequency measurement speed of an analog front-end for a given accuracy. The speed advantages of analog front-end signal processing circuits are known to be an order of 10:1 over pure digital processing circuits [3].

Previous studies assumed that the exact time instants at which the signal switches in frequency are assumed to be known, and the set of candidate hopping frequencies is also assumed known, but Aydh and Polydoros applied coarsely channelized front-end processing, followed by proper post-processing with nonlinear combination of the preprocessor outputs to get practical hop-timing estimators [4]. J. Lehtomäki et al. studied the interception and detection of slow frequency hopping signals using a sweeping channelized radiometer [5]. They concluded that if the number of hops observed per decision is large frequency sweeping decreases the performance.

Our problem is to design a multi-level down-converting super-heterodyne intercept receiver to measure frequencies of all simultaneous intercepted signals in the frequency band $B_t$ with resolution $\delta f$ and 100% intercept probability. It is required to minimize the total frequency measurement time $t_m$ such that the maximum discernible rate of input frequency variation ($R_{\text{max}} = 1/ t_m$) is maximized. The carrier frequency resolution is the bandwidth of the finest level band-pass filter. This excludes all instantaneous spectrum analysis techniques, such as compressive, electro-optical and FFT. Interferometric Frequency Discriminator techniques are also excluded; since they can not work with simultaneous signals. It is assumed that the minimum frequency separation between any two signals is greater than $\delta f/2$, which is a practically viable assumption. Different possible strategies will be discussed and an optimal approach will be concluded. In each case an analytical formula for the frequency measurement time is developed and a constrained optimization problem is formulated and discussed. Finally, a comparison is done on a specific case study.

2. A SIMPLE SCANNING SUPER-HETERODYNE RECEIVER

The simplest solution is the direct scanning of the total frequency band $B_t$ using an IF band-pass filter with $BW = \delta f$ as shown in Fig.1. Such a solution is expected to be too slow.

![Fig. 1. A simple scanning super-Heterodyne receiver](image)
The total measurement time will be:

\[ t_m = (B_t/\delta f).S \]  \hspace{1cm} (1)

where \( S \) is the step period

Since at each frequency step the receiver must remain at least \( t_G \) [sec] to get any existing signal at that frequency, where \( t_G \) is the group delay of the filter; i.e. the minimum time necessary for the band-pass filter to give an output, the minimum step period is:

\[ S_{\text{min}} = t_G \]  \hspace{1cm} (2)

Therefore

\[ t_m \geq (B_t \cdot t_G /\delta f) \]  \hspace{1cm} (3)

Group delay is the derivative of transfer function phase [6]. As frequency resolution gets better, not only \( \delta f \) decreases, but also the filter group delay \( t_G \) increases; since a narrower BW for a realizable filter with limited Q means a lower center frequency. The group delay of a filter increases with the decrease of its center frequency and its bandwidth [7]. Consequently, the total measurement time may go to unacceptable values; exceeding the expected time interval between frequency hops.

### 3. A MULTI-LEVEL SCANNING RECEIVER

We can accelerate the frequency measurement by scanning the frequency band at different IF levels. The total frequency band is divided into \( N_1 = 2^n_1 \) equal frequency steps. These steps are sequentially down-converted to the first IF. The first IF filter with bandwidth \( \Delta f_1 = (B_t/N_1) \) is divided into \( N_2 = 2^n_2 \) equal channels; the bandwidth of each is \( \Delta f_2 = B_t/(N_1N_2) \), and so on. The basic architecture of such a receiver is shown in Fig.2.

Stepping the first local oscillator frequency by \( \Delta f_1 = B_t/N_1 \); subsequent frequency steps are down converted to the first IF. The receiver sequentially investigates each first-level frequency step for signal existence. If after a time interval \( T_{C1} \) of a new step there is no signal; the receiver goes to the next frequency step. The period \( T_{C1} \) should be longer than \( t_{G1} \), where \( t_{G1} \) is the group delay of the first level band-pass filter. It is possible to select \( T_{C1} = t_{G1} + T_R \); where \( T_R << t_{G1} \).

If there is some signal; it stops the first-level frequency scanning while it scans this “active” channel by stepping the second local oscillator to down-convert successive second-level steps to the second IF with bandwidth \( \Delta f_2 = \Delta f_1/N_2 \). The process is repeated at different subsequent levels until the finest frequency resolution \( \Delta f_k = \delta f \). If a signal is found in one of the finest level channels; the whole frequency word is sent to the output frequency register and the finest LO is incremented to check the next finest step for signal existence. When the receiver completes the \( i \)th level scan, it steps the \((i-1)\)th local oscillator to check the next \((i-1)\)th level channel for signal existence.
The process continues until the first local oscillator goes to its last value. The total number of effective frequency steps is:

\[ N_i = \prod_{i=1}^{k} N_i = \frac{B_i}{\delta f} \tag{4} \]

The final frequency word length will be:

\[ \sum_{i=1}^{k} n_i \] \tag{5}

where \( n_i = \log_2 (N_i) \) is the number of bits at the \( i^{th} \) level.

\( k \) is the number of scanning levels.

We can estimate the time required for a three-level scanning frequency measurement process as follows:

\[ t_m = (N_1 - N_{A1})T_{C1} + N_{A1}\cdot [t_{G1} + (N_2 - N_{A2})T_{C2} + N_{A2}\cdot [t_{G2} + (N_3 - N_{A3})T_{C3} + N_{A3} \cdot (t_{G3} + T_R)]] \tag{6} \]

where \( T_R \) is the time interval necessary to read the output register.

\( N_{Ai} \) is the number of active channels at the \( i^{th} \) level, where some signal exists.

It is possible to select at all levels:

\[ T_{Ci} = t_{Gi} + T_R \tag{7} \]

Such a feasible selection will be useful in the comparison between different techniques.
The number of active channels at any level \(i\) should be between 0 and \(N_i\). If we assume a uniform frequency distribution for the existing signals within an active channel at level \(i\) with average probability density \(p_i\), we can assume that:

\[
N_{Ai} = p_i N_i
\]  

(8)

The total time required for frequency measurement becomes:

\[
t_m = (1-p_1)N_1(t_{G1}+T_R) + p_1N_1\{t_{G1} + (1-p_2)N_2(t_{G2}+T_R) + p_2N_2[t_{G2} + (1-p_3)N_3(t_{G3}+T_R)]\}
\]

\[
= N_1t_{G1} + p_1N_1N_2t_{G2} + p_1N_1p_2N_2N_3t_{G3} + [(1-p_1)N_1 + p_1N_1(1-p_2)N_2 + p_1N_1p_2N_2N_3]T_R
\]  

(9)

The process can be extended to \(k\) levels, and a formula for frequency measurement time will take the form:

\[
t_m = N_1t_{G1} + \sum_{j=2}^{k} N_j t_{Gj} \left( \prod_{i=1}^{j-1} p_i N_i \right)
\]

\[+ T_R \left[ (1-p_1)N_1 + \sum_{j=2}^{k} (1-p_j)N_j \left( \prod_{i=1}^{j-1} p_i N_i \right) + N_k \left( \prod_{i=1}^{k-1} p_i N_i \right) \right]
\]  

(10)

where \(T_R < < t_{Gk}\) is clock period used to read a shift register at the last level output.

This time has to be minimized under the following conditions:

1. \(N_t = \prod_{i=1}^{k} N_i \geq \frac{B_t}{\delta f}\)  
   (11)

   where \(B_t\) is the total receiver bandwidth and \(\delta f\) the required frequency resolution.

2. \(B_t / N_1 \leq B_{max}\)  
   (12)

   where \(B_{max}\) is the maximum bandwidth allowed by receiver sensitivity condition

This is a nonlinear programming problem to be solved for the optimum number of levels \(k\) and the optimum number of channels at each level \(\{N_i\}\).

It can be noticed from (10) that the longest filter group delay \(t_{Gk}\) is multiplied with the product of all numbers of active channels at all levels; which leads to very long measurement times in case of high signal density environments.

### 4. CHANNELIZING THE RECEIVER

Another solution is to channelize the receiver. A channelized receiver has a total frequency field of view \(B_t\) divided into a total number of effective channels \(N_{eff}\) at \(k\) subsequent levels.
The term “effective channels” means that not all the \( N_{\text{eff}} \) channels must co-exist in parallel at the same time; giving chances for band folding and/or queuing channelization. Two basic design requirements for a channelized receiver are the total field of view \( B_t \) and the final frequency resolution \( \delta f \). Channelization strategy means how to select the number of levels \( k \) and the set of numbers of channels \( \{N_i\} \) to minimize a certain objective function under a set of practical constraints; including the two above mentioned design requirements. In this study, the objective function to be minimized is the frequency measurement time \( t_m \).

The channelized receiver sensitivity constraint is formulated such that the minimum detectable signal should be lower than the input power level to be received from the lowest expected transmitted power at the longest expected range. This results in an upper limit on the instantaneous receiving bandwidth at the first detection level, which can be formulated as follows:

\[
S_{\text{min}} [dBM] = -114 + 10 \log \left( \frac{B_e}{1 \text{MHz}} \right) + NF + SNR < S_{\text{r min}} [dBM]
\]

\[
S_{\text{r min}} [mw] = K.T.B_e \cdot F. \left( \frac{S}{N} \right)_{\text{min}} < S_{\text{r min}} [mw]
\]

\[
B_{\text{max}} = \frac{S_{\text{r min}} [mw]}{K.T.F. (S/N)_{\text{min}}}
\]

where

\[
S_{\text{r min}} [mw] = \left( \frac{P_{\text{r min}}[mw] \cdot G_{\text{r min}}}{4\pi R_{\text{max}}^2} \right) \left( \frac{k_p G_s \lambda^2}{4\pi} \right)
\]

\( S_{\text{min}} \) = receiver sensitivity
\( B_e \) = receiver effective noise bandwidth \( \approx \) channel bandwidth at the first detection level.
\( NF \) = receiver noise figure [dB] = 10log(F)
\( SNR \) = minimum Signal-to-Noise Ratio [dB] for reliable signal detection = 10log(S/N)
\( S_{\text{r min}} \) = smallest expected signal at Rx input
\( P_T \) = transmitted power
\( G_T \) and \( G_R \) = transmitting and receiving antenna gains, respectively
\( K_p \) = polarization match factor < 1
\( \lambda \) = wavelength

Two possible strategies for receiver channelization will be. Band folding will not be considered due to its inherent ambiguity that makes it impractical.

**Full Channelization**

A brute force solution is to divide each first-level channel into \( N_2 \) second-level channels; each of which into \( N_3 \) third-level channels and so on. Further finer levels are added until the last \( k^{\text{th}} \)
level gives the required frequency resolution $\delta f$. The total number of physical channels $N_i$ is the product of $N_i$ on the $k$ levels, as given by (14).

$$
\Delta f_i = \frac{B_i}{N_i}
$$

$$
\Delta f_i = \frac{B_{i+1}}{N_{i+1}} \left( \frac{B_i}{\prod_{i=1}^{i} N_i} \right) = \frac{B_i}{\prod_{i=1}^{i} N_i}
$$

$$
\Delta f_i = \delta f
$$

$$
N_i = N_{eff} = \prod_{i=1}^{i} N_i = \frac{B_i}{\delta f}
$$

The frequency measurement time will be very short; given that all channels at all levels are simultaneously activated:

$$
t_m = \sum_{i=1}^{k} t_{Gi} + T_{Ci} N_k
$$

Where $T_{Ck}$ is the clock used for reading the $k^{th}$ level output status = $T_R$

$N_k$ is the number of channels at the $k^{th}$ level.

Although this solution satisfies all the above mentioned design requirements, it is impractically expensive. It needs excessive volume, size, weight and number of hardware components.

**Queuing Channelization**

In this technique the total number of physical channels is given by:

$$
N_{ph} = \sum_{i=1}^{k} N_i
$$

while the total effective number of channels remains $N_{eff} = (B_i/\delta f) = \prod_{i=1}^{k} N_i$, as given in (14).

The main advantage of this technique is that it uses much less hardware than full channelization; since $N_{ph} \ll N_{eff}$. The cost is a relatively longer measurement time which will be analyzed and minimized.

Starting from the multi-level scanning receiver, instead of stepping the local oscillator at each level $i$ by $\Delta f_i$ and waiting $t_{Gi}$ after each step to decide if the channel is active, the input signal at each level $i$ is coupled to all the $N_i$ channels simultaneously. After only one $t_{Gi}$ period, the outputs of all active channels at the $i^{th}$ level are simultaneously detected by independent threshold detectors and a fast digital scan with clock period $T_R << t_{Gi}$ extracts their outputs to determine which channels at this level are active. The detectors outputs are temporarily stored in a shift register with length $N_1$. The measurement logic scans this shift register at a
clock rate $R_R = 1/T_R$ to get the status of each channel at this level. Only active channels at level $i$ are sequentially down converted to the $(i+1)^{th}$ level. The $k^{th}$ level channel bandwidth is $\Delta f_k = \delta f$. The basic architecture of level $i$ of such a receiver is shown in Fig.3.

**Frequency measurement time at two channelization levels:**

Let us start with two channelization levels. The first level has $N_1$ parallel filters that take time $t_{G1}$ to give their outputs. The outputs of those filters are simultaneously compared to detection threshold by $N_1$ simultaneous threshold detectors. The outputs of those detectors are temporarily stored in a shift register with length $N_1$. The measurement logic scans this shift register at a certain clock rate $R_{C1} = 1/T_{C1}$ to get the status of each channel at this level. Each active channel at the first level is down converted to IF where it goes to $N_2$ different filters at the second level, whose outputs; after a delay time $t_{G2}$, are simultaneously compared to a detection threshold. The detected outputs are temporarily stored in another shift register with length $N_2$. The measurement logic scans the second shift register at a clock rate $R_{C2} = (1/T_R)$ to get the status of each channel at the second level. We can write an expression for the measurement time as follows:

$$t_m = t_{G1} + N_{A1}(t_{G2} + T_RN_2) + (N_1 - N_{A1})T_R \quad (17)$$

Substituting from (7)

$$t_m = t_{G1} + p_1N_1(t_{G2} + T_RN_2) + (1 - p_1)N_1T_R$$

$$= t_{G1} + p_1N_1t_{G2} + [p_1N_1N_2 + (1 - p_1)N_1]T_R \quad (18)$$

where $p_1 = probability \ density \ of \ signal \ existence \ within \ an \ active \ channel \ at \ the \ 1^{st} \ channelization \ level.$
Generalizing the queuing solution

Applying the queuing solution at 3 levels

\[ t_m = t_{G1} + p_1N_1t_{G2} + p_1N_1p_2N_2t_{G3} + [p_1N_1p_2N_2N_3 + p_1N_1(1-p_2)N_2 + (1-p_1)N_1]T_R \]

If we apply the queuing solution at \( k \) levels; the total expected time required to measure all co-existing frequencies in the band of interest will be:

\[ t_m = t_{G1} + \sum_{j=2}^{k} t_{Gj} \left( \prod_{i=1}^{j-1} p_i N_i \right) + T_R \left( (1-p_1)N_1 + \sum_{j=2}^{k} (1-p_j)N_j \left( \prod_{i=1}^{j-1} p_i N_i \right) + N_k \left( \prod_{i=1}^{k-1} p_i N_i \right) \right) \]

(19)

This time has to be minimized; subject to the same constraints given by equations (11) and (12). It is still a nonlinear programming problem.
5. OPTIMIZING THE DESIGN STRATEGY

Comparing the above mentioned techniques; we can easily exclude simple scanning due to its long measurement time and full channelization due to its evident complexity. We have to compare multi-level scanning with queuing channelization to select one of them as an optimal strategy. The objective function to be minimized is the frequency measurement time $t_m$ and the constraints are those given in (11) and (12). The set of variables to be optimized is the number of levels and the number of channels at each level $\{N_i\}$. A k-level queuing channelized receiver measures frequency within a time $t_{mq}$ given by (19). Comparing it with the frequency measurement time $t_{ms}$ of a k-level scanning receiver given by (10), it is smaller by $(t_{ms} - t_{mq})$ given by:

$$t_{ms} - t_{mq} = (N_1 - 1)t_{Gi} + \sum_{j=2}^{k} (N_j - 1)t_{Gj} \prod_{i=1}^{j-1} p_i N_i$$ (20)

Since $t_{ms} - t_{mq}$ is always positive, queuing channelization can be considered the best intercept receiver design strategy for minimum frequency measurement time under the given constraints. It can also be noted that the measuring time difference between the two techniques increases with the increase of signal density at different levels $\{p_i\}$. The effect of signal distribution on the frequency axis can be the subject of a separate study.

Looking at the objective function equation (19), we can notice the following:

1. The second term determines the order of magnitude of the objective function $t_m$; since $t_{Gi} \gg t_R$ at all levels.

2. The last level group delay $t_{Gk}$ is the most significant parameter affecting $t_m$ since:
   a. it is multiplied with the product of all $N_i$ except $N_k$
   b. $t_{Gi} > t_{Gj}$ for each $i > j$

   Therefore, the last level must get the highest number of channels.

3. The sensitivity constraint (12) forces a lower limit on $N_1$ given by $(B_i / B_{max})$. If we note that $N_1$ is a factor of most of the significant terms of (19); it is evident that this constraint has to be carefully got around. This will be analyzed in the next paragraph.

Getting around the sensitivity constraint

The idea is to start with scanning at the first level without detection. It is not necessary to detect signals at the first channelization level. Signal detection may start at any channelization level $d$. The channel bandwidth at level $d$ is given by:

$$BW_d = \frac{B_i}{\prod_{i=1}^{d} N_i}$$ (21)

The sensitivity constraint can be written as:

$$\prod_{i=1}^{d} N_i \geq \frac{B_i}{B_{max}}$$ (22)
where \( N_i \) is the number of channels at the \( i^{th} \) channelization level
\( d_i \) is the order of the first detection level
\( B_i \) is the total frequency field of view
\( B_{\text{max}} \) is the maximum allowable detection bandwidth satisfying sensitivity condition.

This constraint means that the first detection level should not permit a channel wider than \( B_{\text{max}} \) [MHz]. All previous channelization levels violating this condition can be covered by local oscillator scanning and down conversion without detection. Since the first detection level satisfies the condition (22), all next channelization levels will satisfy it; since
\[
\prod_{i=1}^{d_i} N_i = \prod_{i=1}^{d_i} N_i \prod_{i=d_i}^{d_j} N_j \geq \prod_{i=1}^{d_j} N_j \quad \forall j \geq d_i
\] (23)

**A modified queuing channelized receiver with a first scanning stage**

Such a **hybrid receiver** design strategy may give better results. It avoids the relatively large number of channels at the first stage resulting from sensitivity constraint. Instead of the first IF BPF there is a bank of \( N_2 \) parallel band-pass filters; each with BW \( \Delta f_2 = B_t/(N_1 N_2) \). The sensitivity constraint applies to \( N_1 N_2 \) instead of \( N_1 \) such that \( N_1 N_2 \geq (B_t/B_{\text{max}}) \). The receiver scans the total frequency band \( B_t \) at a number of steps \( N_1 \). At each step it waits a time interval \((t_{G2} + t_R)\). If any second-level channel is active, the receiver stops the first-level scan, reads the second-level output status and successively down-converts each second-level active channel to the second IF where detection and queuing take place. All next levels are queuing channelized. The frequency measurement time of such a four-level hybrid receiver can be expressed as follows:

\[
t_{\text{mqm}} = (1-p_1)N_{1m}(t_{G2} + t_R) + p_1 N_1 \{t_{G2} + p_2 N_2[t_{G3} + p_3 N_3(t_{G4} + N_4 T_R)] + (1-p_3)N_3 T_R \} + (1-p_2)N_2 T_R \]

\[= N_1 t_{G2} + p_1 N_1 p_2 N_2 t_{G3} + p_1 N_1 p_2 N_2 p_3 N_3 t_{G4} + T_R[(1-p_1)N_1 + p_1 N_1 (1-p_2)N_2 + p_1 N_1 p_2 N_2 (1-p_3)N_3 + p_1 N_1 p_2 N_2 p_3 N_3 N_4]\]

If we compare it with that of a queuing channelized solution (9) the difference will be:

\[
t_{\text{mq}} - t_{\text{mqm}} = t_{G1} + (p_1 N_1 - N_{1m})t_{G2} + (N_1 - N_{1m})[p_1 p_2 N_2 t_{G3} + p_1 p_2 N_2 p_3 N_3 t_{G4} + ((1-p_1) + p_1 (1-p_2) N_2 + p_1 p_2 N_2 (1-p_3) N_3 + p_1 p_2 N_2 p_3 N_3 N_4]T_R(N_1 - N_{1m})
\]

Since \( N_{1m} < N_1 \); the modified solution will have a shorter measurement time. The difference will increase with increased signal density. The frequency measurement time of a k level hybrid receiver will be:

\[
t_m = N_1 t_{G2} + \sum_{j=3}^{k} t_{Gj} \left( \prod_{i=1}^{j-1} p_j N_i \right) + T_k \left[ (1-p_1)N_1 + \sum_{j=2}^{k} (1-p_j) N_j \left( \prod_{i=1}^{j-1} p_j N_i \right) \right]
\] (24)
6. A CASE STUDY

Let us design an intercept receiver to measure frequency hopping signals in the VHF band for ground tactical communications with resolution $\delta f = 25$ kHz and 100% intercept probability. It is required to minimize the total frequency measurement time $t_m$ such that the maximum discernible rate of input frequency variation ($R_{\text{max}} = 1/ t_m$) is maximized. Since $B_t = 60$ MHz and $\delta f = 25$ kHz; the total effective number of channels is $B_t/\delta f = 2400$. A synthesis-simulation study by the GENESYS CAD software resulted in the following values of group delay $t_G$ for fifth-order Bessel filters.

<table>
<thead>
<tr>
<th>Table-1</th>
<th>Group delay of typical 5th order Bessel band-pass filters</th>
</tr>
</thead>
<tbody>
<tr>
<td>BW [MHz]</td>
<td>0.025</td>
</tr>
<tr>
<td>$f_c$ [MHz]</td>
<td>0.262</td>
</tr>
<tr>
<td>$Q = f_c/BW$</td>
<td>10.48</td>
</tr>
<tr>
<td>Insertion Loss [dB]</td>
<td>3</td>
</tr>
<tr>
<td>$t_G$ [$\mu$s]</td>
<td>30</td>
</tr>
</tbody>
</table>

In each case an appropriate value was chosen for the center frequency $f_c$ to guarantee a practical value for the filter Q factor [$Q = f_c/BW$] and consequently, a 3 dB insertion loss. Other filter design shapes; such as Chebyshev and Butterworth, gave longer group delays. The following table summarizes the results of applying the different design strategies that have been studied.

<table>
<thead>
<tr>
<th>Table-2</th>
<th>Evaluation results of different design strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver Design Strategy</td>
<td>$N_1$</td>
</tr>
<tr>
<td>---------</td>
<td>--------</td>
</tr>
<tr>
<td>Simple scanning superhet</td>
<td>2400</td>
</tr>
<tr>
<td>3-level scanning</td>
<td>60</td>
</tr>
<tr>
<td>four-level scanning</td>
<td>60</td>
</tr>
<tr>
<td>fully channelized</td>
<td>60</td>
</tr>
<tr>
<td>three-level queueing channelized</td>
<td>60</td>
</tr>
<tr>
<td>four-level modified (hybrid) queueing channelized</td>
<td>5</td>
</tr>
</tbody>
</table>
7. CONCLUSION

The main outlines of the proposed optimal design strategy are the following:

1. Determine the required frequency band and resolution and apply (14) to get the total effective number of channels.
2. Apply the sensitivity constraint (12) to get the lower limit of $N_1$. If it is apparently a big number use scanning without detection at the first level and let the product $N_1N_2$ equal this limit. Try to select $N_1 < N_2$ and satisfy this condition.
3. Design a Bessel BPF with $BW_1 = \Delta f_1/ N_1$ and compute its group delay $t_{G1}$.
4. Repeat the procedure for the second level in case of applying the modified strategy.
5. Determine the objective function to be minimized $t_m$ according to (19) or (24) and substitute $N_1$ and $t_{G1}$ ($N_1$, $N_2$, $t_{G1}$, and $t_{G2}$). Assume that $p_i = 0.5$ at all levels, if the actual signal distribution is not exactly known.
6. Construct different chromosomes, each with a certain combination of $k$ and $\{N_i, \Delta f_i, t_{Gi}\}$ and apply genetic techniques to arrive at the optimal one. In each chromosome always let $N_i < N_j$ for all $i < j$.

8. REFERENCES


Nomenclatures:

$t_m$ $\{N_i\}$, $\delta f$, $\Delta f_i$, $t_{Gi}$, $T_R$, $p_i$, $B_{max}$, $k$