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Interference Effect on the Post-Correlation Carrier to Noise Ratio for GPS Receiver

By

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Abstract:

Interference is one of the major concerns in using the global positioning system (GPS) for civilian and military applications. In this paper, a closed form analytical expressions for the continuous wave interference (CWI), and matched spectrum interference (MSI) power at correlator output are derived. The post-correlation carrier to noise ratio (C/No) for the GPS L1 coarse acquisition (C/A) signal is analytically derived in the presence of CWI, and MSI. The effect of the GPS correlator coherent integration time is also investigated under the impact of these interferences. The derived analytical formulas are verified by extensive the computer based simulation result**.**

Keywords:

Interference signals, GPS receiver, GPS correlator output power, coherent integration time.

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1. Introduction:

The Global positioning system (GPS) is a satellite-based worldwide all weather navigation and timing system[1, 2]. This system was developed by the Department of Defense to support the military forces of the United States of America by providing world-wide, real-time positions. GPS can be used for civilian applications even though it was developed for military applications[2].
The interference signals can be considered as one of the most disruptive events in the operation of a GPS receiver, as the interference signals affect the operation of the automatic gain control (AGC) and the low noise amplifier (LNA) in the GPS receiver frontend, along with the acquisition and carrier-code tracking loops are also affected [3].
The interference effect on the GPS system has already been addressed by many researchers. Betz, J. W, et al [1, 4] suggest an analytical model to interpret the post-correlation effects of interference signals, resulting in very useful mathematical expressions for the carrier to noise ratio (C/No), the code tracking error and the carrier tracking error, under the assumption continuous spectrum density functions and neglect the effect of Doppler frequency, there is a limitation to interpret the effects of CWI signals. In [5] the effects of three different types of interference; CWI, pulse CW, and swept CW were studied on the C/No of the received GPS signal. In [6] the CWI effects on the tracking performance of the GPS receivers is investigated.
In this paper, a mathematical expression for the CWI and MSI correlator output power is derived. The post-correlation (C/No) in the presence of interference signal is also studied.
This paper is organized as follows. The carrier to noise ratio C/No in presence of interference signal is mentioned in section II. In section III, the correlation output power for the GPS signal is derived. In section IV, the correlation output power and C/No for the CWI is derived. The post-correlation C/No in the presence of MSI signal is derived in section V. In section VI, the set of numerical and simulation results for different case is presented. Section VII concludes the paper.

2. The Carrier to Noise Ratio in The Presence of Interference:

The ratio of the received GPS signal power level to the noise power level in a 1 Hz bandwidth is called the carrier-to-noise density ratio C/No. The estimation of the C/No is essential, as it assesses or predicts the receiver performance under the impact of interference signals. From[7] the correlation filter bandwidth equals to reciprocal of integration time $(1/T_d)$, and it take value from 50 to 1000Hz, because the coherent integration time take value from 1 to 20ms, then the interference power density at correlator output in the unit of [W/Hz] can be written as

$$N_I = P_I . T_d \tag{1}$$

where P_I is the correlator output power for the interference signal. The post-correlation carrier to noise ratio C/No can be expressed as

$$C/N_o = \frac{P_G}{(N_o)_{post}} \tag{2}$$

where P_G is the GPS signal power at the correlator output, and $(N_o)_{post}$ is the post-correlation noise density in presence of interference signal and AWGN, can be defined as below

$$(N_o)_{post} = N_o + N_I \tag{3}$$

Substituting equation (3) into (2) gives, the post-correlation carrier to noise ratio C/No in the presence of interference and AWGN.

$$C/N_o = \frac{P_G}{N_o + N_I} = \frac{P_G}{N_o + T_d P_I} \tag{4}$$

From (4), it can be seen, the interference signal raises the noise floor in the correlator output causing a drop in the C/No.

3. The GPS Signal Power at the Correlator Output:

The correlator plays a vital role in the signal processing system of any GPS receiver, because its output is fed into the acquisition unit, code tracking loop, and carrier tracking loop.
The signal at the output of the GPS receiver front-end in presence of interference and AWGN can be written as

$$r(nT_s) = r(nT_s) + I(nT_s) + n(nT_s) \tag{5}$$

The input signal $r(nT_s)$ is applied at the correlator input, the carrier and code is wiped off by multiplying the $r(nT_s)$ by reference carrier at frequency $\hat{f}_D$ and phase $\hat{\theta}$, and then multiplied by a local replica code $C(nT_s - \tau_n)$, where τ_n is reference C/A code time delay. The resulting signals are then coherently integrated within an integration time $T_d = N_s . T_s$, where N_s is the number of samples in the integration time, T_s is the sampling period. Then the output of the integrator can be written as

$$A(\tau_n, \hat{f}_D) = \frac{1}{N_s} \sum_{n=0}^{N_s-1} r(nT_s)\, C(nT_s + \tau_n)\, e^{j(2\pi \hat{f}_D n T_s + \hat{\theta})}. \tag{6}$$

In order to obtain the correlator output power, the correlator amplitude is passed through the square and absolute block, and then the correlator output power can be written as

$$P_c(\tau_n, \hat{f}_D) = \left| \frac{1}{N_s} \sum_{n=0}^{N_s-1} r(nT_s)\, C(nT_s + \tau_n)\, e^{j(2\pi \hat{f}_D n T_s + \hat{\theta})} \right|^2. \tag{7}$$

In order to increase the correlation gain, *K* instances of the output of the correlator are summed in a noncoherent integration block[3, 8]. The noncoherent correlator output power is given by

$$P_{nc}(\tau_n, f_s) = \sum_{k=0}^{K-1} \left| \frac{1}{N_s} \sum_{n=0}^{N_s-1} r(nT_s)\, C(nT_s - \tau_n)\, e^{j(2\pi \hat{f}_D nT_s + \hat{\theta})} \right|^2, \tag{8}$$

where *K* indicates a noncoherent integration factor, the output of coherent integration is independent of *K*. Finally the noncoherent correlator output power can be written as

$$P_{nc}(\tau_n, f_s) = K. \left| \frac{1}{N_s} \sum_{n=0}^{N_s-1} r(nT_s)\, C(nT_s - \tau_n)\, e^{j(2\pi \hat{f}_D nT_s + \hat{\theta})} \right|^2. \tag{9}$$

Since all operations in the correlation block are linear[3], the three components at the correlator input (useful GPS signal $s(nT_s)$, interference $I(nT_s)$, noise $n(nT_s)$) can be investigated separately. Let us first consider the scenario in which the correlator output power is analyzed for the GPS satellite signal without including the effect of interference and noise. In this case the signal at the correlator input $r(nT_s)$ equals to the GPS signal $s(nT_s)$, which it can be written as[1]

$$s(nT_s) = \sqrt{P_s} D(nT_s - \tau_s) C(nT_s - \tau_n) e^{j(2\pi f_D nT_s + \theta_s)} \tag{10}$$

where P_s is the GPS signal power, $D(nT_s)$ is the navigation data, $C(nT_s)$ is the satellite C/A code, f_D is the satellite GPS signal frequency, and θ_s is the carrier phase.

Assuming, there is no navigation data transmission during the coherent integration process $D(nT_s - \tau_s) = 1$ [2], and substituting (10) into (9), then correlator output power for the GPS signal can be written as

$$\begin{aligned} P_G(\tau_n, \Delta f_D) &= K \left| \frac{1}{N_s} \sum_{n=0}^{N_s-1} \sqrt{P_s} C(nT_s - \tau_s)\, C(nT_s - \tau_n)\, e^{j(2\pi \Delta f_D nT_s + \Delta\theta)} \right|^2 \\ &= KP_s \left| \frac{1}{N_s} \sum_{n=0}^{N_s-1} C(nT_s)\, C(nT_s + \Delta\tau) e^{j(2\pi \Delta f_D nT_s + \Delta\theta)} \right|^2, \end{aligned} \tag{11}$$

where $\Delta f_D = f_D - \hat{f}_D$ is the carrier frequency estimation error, the code phase estimation error is $\Delta\tau = \tau_s - \tau_n$, and $\Delta\theta = \theta_s - \hat{\theta}$ is the phase error.

The multiplication of C/A code with the delayed version of that code generates a new code with same period T_c.

$$C_m(nT_s) = C(nT_s)\, C(nT_s + \Delta\tau) \tag{12}$$

where $C_m(nT_s)$ represents a new code that is generated by multiplication of the C/A code $C(nT_s)$ by its delayed version. Then the mathematical expression for the GPS signal power at the correlator output can be written as

$$P_G(\tau_n, \Delta f_D) = KP_s \left| \frac{1}{N_s} \sum_{n=0}^{N_s-1} C_m(nT_s + \Delta\tau)\, e^{j(2\pi\Delta f_D nT_s + \Delta\theta)} \right|^2 \tag{13}$$

The periodic $C_m(nT_s)$ code can be written as

$$C_m(nT_s) = \sum_{i=-N_c}^{N_c} \acute{m}_i . e^{j2\pi\frac{i}{T_c}nT_s} \tag{14}$$

where $\acute{m}_i$ is the normalized Fourier series coefficients of the $C_m(nT_s)$code.
Substituting (14) into (13), the GPS signal power at the correlator output can be written as

$$\begin{aligned} P_G(\Delta\tau, \Delta f_D) &= P_s K \left| \sum_{i=-N_c}^{N_c} \acute{m}_i . \frac{1}{N_s} \sum_{n=0}^{N_s-1} e^{j\left(2\pi\left[\frac{i}{T_c}+\Delta f_D\right]nT_s\right)} \right|^2 . \left|e^{j(\Delta\theta)}\right|^2 \\ &= P_s K \left| \sum_{i=-N_c}^{N_c} \acute{m}_i . \mathrm{sinc}(\pi\left[\frac{i}{T_c} + \Delta f_D\right]T_d) \right|^2 \end{aligned} \tag{15}$$

when the GPS receiver is operating in its tracking mode. Under this assumption, the receiver's reference signal is perfectly aligned with the satellite signal with small code phase and carrier estimation error $(\Delta\tau \ll T_a, \Delta f \ll 1/T_c)$. In this case, substitute $i = 0$, and $|\acute{m}_0|^2 = \left(1 - \frac{|\Delta\tau|}{T_a}\right)^2$ in (15), then the mathematical expression for the GPS signal power at the correlator output when the GPS receiver is operating in its tracking mode can be written as

$$P_G(\Delta\tau, \Delta f_D) = P_s K \left(1 - \frac{|\Delta\tau|}{T_a}\right)^2 (\mathrm{sinc}(\pi\Delta f_D T_d))^2 \tag{16}$$

4. CWI at the correlator output:

The processing time in the correlator is equal to the integration time T_d, the frequency resolution (difference between two spectrum components) equals $1/T_d$. Thus, the interference signal is considered as CWI if its bandwidth is less than or equal to the reciprocal of the integration time$\left(B_N \le \frac{1}{T_d}\right)$, where B_N is the interference bandwidth. The CWI signal at the input of the correlator can be modeled as

$$I_{cw}(nT_s) = \sqrt{P_j}\ . e^{j(2\pi f_j nT_s + \theta_j)}, \tag{17}$$

Where P_j is the interference signal power, f_j is the CWI frequency at the input of the correlator, and θ_j is the interference phase.
The correlator output power for the CWI signal using (9) and (17) can be written as

$$P_{CW}(\tau_n, \Delta f_j) = K\left|\frac{1}{N_s}\sum_{n=0}^{N_s-1}\sqrt{P_j}\, C(nT_s - \tau_n)\, e^{j(2\pi\Delta f_j nT_s + \Delta\theta_j)}\right|^2 \quad (18)$$

Where the interference frequency error Δf_j is the difference between the interference frequency f_j and the receiver reference carrier frequency $\Delta f_j = f_j - \hat{f}_D$, and $\Delta\theta_j$ is the phase error.

The periodic C/A code , $C(t)$, can be written as

$$C(t) = \sum_{i=-N_c}^{N_c} \acute{c}_i e^{j2\pi\frac{i}{T_c}t}, \quad (19)$$

where $\acute{c}_i$ is the Fourier series coefficients. The spectrum of the reference GPS and CWI signals is shown in Figure 1.

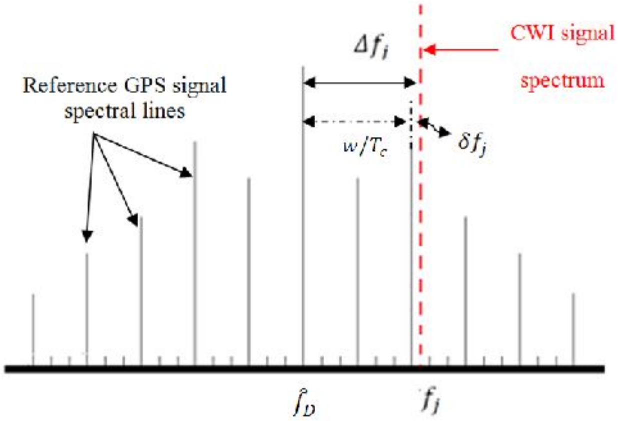

Figure 1: The reference GPS and CWI signals spectrum.

Substituting (19) into (18), the CWI power at the correlator output can be written as

$$\begin{aligned} P_{CW}(\tau_n, \Delta f_j) &= P_j K\left|\sum_{i=-N_c}^{N_c} \acute{c}_i e^{-j2\pi\frac{i}{T_c}nT_s} e^{-j2\pi\frac{\tau_n}{T_c}i} \cdot \frac{1}{N_s}\sum_{n=0}^{N_s-1} e^{j(2\pi\Delta f_j nT_s)}\right|^2 \cdot \left|e^{j(\Delta\theta_j)}\right|^2 \quad (20) \\ &= P_j K\left|\sum_{i=-N_c}^{N_c} \acute{c}_i e^{-j2\pi\frac{\tau_n}{T_c}i} \cdot \frac{1}{N_s}\sum_{n=0}^{N_s-1} e^{j\left(2\pi\left[\frac{i}{T_c}+\Delta f_j\right]nT_s\right)}\right|^2 \\ &= P_j K\left|\sum_{i=-N_c}^{N_c} \acute{c}_i e^{-j2\pi\frac{\tau_n}{T_c}i} \cdot \mathrm{sinc}\left(\pi\left[\frac{i}{T_c}+\Delta f_j\right]T_d\right)\right|^2 \end{aligned}$$

From Figure 1, the interference frequency error equals to $\Delta f_j = f_j - f_D = \frac{w}{T_c} + \delta f_j$, where w is the frequency difference between the CWI signal and the reference signal as integer multiple of reciprocal of the C/A period (multiple integer number of one KHz), and δf_j is the difference between the interference frequency and the nearest spectral line in the reference signal spectrum. The mathematical expression for the CWI power at the correlator output can be written as

$$P_{CW}(\tau_n, w, \delta f_j) = P_j K \left| \sum_{i=-N_c}^{N_c} \acute{c}_i e^{-j2\pi \frac{\tau_n}{T_c} i} . \mathrm{sinc}(\pi \left[\frac{i}{T_c} + \frac{w}{T_c} + \delta f_j \right] T_d) \right|^2 \tag{21}$$

From (21), the main lobe of the sinc function $\left(sinc(\pi \left[\frac{i}{T_c} + \frac{w}{T_c} + \delta f_j\right] T_d)\right)$ has width equal to $1/T_d$, and the integration time is greater than or equal to the C/A code period ($T_d \geq T_c$), for that if $i \neq -w$ then the interference frequency is outside the main lobe. The CWI power at the correlator output has a value near zero when $i \neq -w$. Finally, the mathematical expression for the CWI power at the correlator output can be written as

$$P_{CW}(w, \delta f_j) = P_j . K. |\acute{c}_w|^2 . (\mathrm{sinc}(\pi \delta f_j T_d))^2 \tag{22}$$

Where $|\acute{c}_w|$ is the amplitude of the C/A code spectral-line number w.
The post-correlation C/No in the presence of CWI assess the impact of CWI on the GPS receiver performance, substituting equation (22) and (16) into (4) yields the expression for post-correlation C/No in the presence of CWI when GPS receiver is operating on its tracking mode

$$C/N_o = \frac{P_G}{N_o + P_{CW}} = \frac{P_s K \left(1 - \frac{|\Delta\tau|}{T_a}\right)^2 (\mathrm{sinc}(\pi \Delta f_D T_d))^2}{N_o + T_d . K. P_j . |\acute{c}_w|^2 . (\mathrm{sinc}(\pi \delta f_j T_d))^2} \tag{23}$$

The CWI effect on GPS receiver is corresponding to the amplitude of the nearest C/A code spectral line and integration time.

5. The matched spectrum interference at the correlator output:

The MSI uses the same GPS signal code, modulation, and chipping rate, where in this approach the interference has the same spectral characteristics as the GPS signal transmitted by the satellite system [2]. The reference GPS and MSI signal spectrum is shown in Figure2.

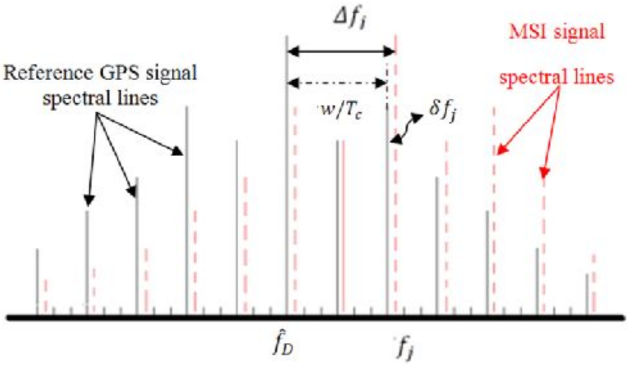

Figure2: The reference GPS and MSI signal spectrum.

The MSI at the input of the correlator is defined as

$$I_{MS}(nT_s) = \sqrt{P_j}.\, P(nT_s - \tau_p).\, e^{j(2\pi f_j nT_s + \theta_j)}, \tag{24}$$

where P_j is the MSI power, $P(nT_s)$ is the MSI C/A spreading code, τ_p is the MSI code delay, f_j and θ_j are the interference frequency and phase, respectively. The correlator output power for the MSI signal is written as

$$P_{MS}(\Delta\tau_p, \Delta f_j) = K \left| \frac{1}{N_s} \sum_{n=0}^{N_s-1} \sqrt{P_j}.\, P(nT_s)\, C(nT_s + \Delta\tau_p)\, e^{j(2\pi\Delta f_j nT_s + \Delta\theta_j)} \right|^2 \tag{25}$$

Where $\Delta\tau_p$ is the MSI code phase error $\Delta\tau_p = \tau_s - \tau_p$. The periodic $P(nT_s)$ code can be expressed as

$$P(nT_s) = \sum_{i=-N_c}^{N_c} p_i e^{j2\pi \frac{i}{T_c} nT_s}, \tag{26}$$

where p_i is the Fourier series coefficients of $P(nT_s)$. The MSI correlator output power can be written as

$$P_{MS}(\Delta\tau_p, \Delta f_j) = K.\, P_j \left| \frac{1}{N_s} \sum_{n=0}^{N_s-1} \sum_{i=-N_c}^{N_c} \sum_{q=-N_c}^{N_c} p_i c_q e^{j2\pi \frac{\Delta\tau_p}{T_c} q} e^{j\left(2\pi\left(\frac{i+q}{T_s} + \Delta f_j\right) nT_s + \Delta\theta_j\right)} \right|^2 \tag{27}$$

let $m = i + q$, then the MSI correlator output power is

$$P_{MS}(\Delta\tau_p, \Delta f_j) = K.\, P_j \left| \sum_{m=-N_c+q}^{N_c+q} \sum_{q=-N_c}^{N_c} \left(p_{m-q} c_q e^{j2\pi \frac{\Delta\tau_p}{T_c} q} \right) . \frac{1}{N_s} \sum_{n=0}^{N_s-1} e^{j\left(2\pi\left[\frac{m}{T_c} + \Delta f_j\right] nT_s\right)} \right|^2 \tag{28}$$

The time-frequency cross-correlation function , $R_c(\Delta\tau_p, m)$can expressed as

$$R_c(\Delta\tau_p, m) = \sum_{q=-N_c}^{N_c} p_{m-q} c_q e^{j2\pi \frac{\Delta\tau_p}{T_c} q}, \tag{29}$$

Substituting in (28). The MSI signal power at the output of the correlator is

$$P_{MS}(\Delta\tau_p, \Delta f_j) = K.\, P_j \left| \sum_{m=-N_c+q}^{N_c+q} R_c(\Delta\tau_p, m) . \mathrm{sinc}\left(\pi \left[\frac{m}{T_c} + \Delta f_j\right] N_s T_s\right) \right|^2 \tag{30}$$

Substitute $\Delta f_j = \frac{w}{T_c} + \delta f_j$ in (30) then the MSI signal power at the output of the correlator is rewritten as

$$P_{MS}(\Delta\tau_p, \Delta f_j) \approx \begin{cases} K.\, P_j \left| R_c(\Delta\tau_p, -w).\, \mathrm{sinc}(\pi \delta f_j N_s T_s) \right|^2 & , m = -w \\ 0 & , m \neq -w \end{cases} \tag{31}$$

Note that the time-frequency cross-correlation function, $R_c(\Delta\tau_p, m)$ is an even function, then the MSI signal power at the output of the correlator can be rewritten as

$$P_{MS}(\Delta\tau_p, w, \delta f_j) = K.P_j|R_c(\Delta\tau_p, w).\operatorname{sinc}(\pi\delta f_j T_d)|^2 \quad (32)$$

The time-frequency cross-correlation function depends on both w (the interference frequency error as multiple integer number of KHz), and $\Delta\tau_p$ (the MSI code phase error), thus $R_c(\Delta\tau_p, w)$ can be written as

$$R_c(\Delta\tau_p, w) = \frac{1}{N_s}\sum_{n=0}^{N_s-1} P(nT_s)\, C(nT_s + \Delta\tau_p)\, e^{j2\pi\frac{w}{T_c}nT_s} \quad (33)$$

the multiplication of the MSI C/A spreading code $P(nT_s)$ with the GPS receiver reference code $C(nT_s + \Delta\tau_p)$ in the correlator yields another code call a crosscorrelation sequence (CCS) $G(nT_s + \Delta\tau_p)$, which is no longer a Gold code yet, but it has the same period.

$$G(nT_s) = C(nT_s + \Delta\tau_p)P(nT_s) \quad (34)$$

Then the time-frequency cross correlation is written as

$$R_c(\Delta\tau_p, w) = \frac{1}{N_s}\sum_{n=0}^{N_s-1} G(nT_s)\, e^{j2\pi\frac{w}{T_c}nT_s} \quad (35)$$

The periodic cross-correlation sequence $G(nT_s)$ can be written as

$$G(nT_s) = \sum_{i=-N_g}^{N_g} \grave{g}_i . e^{j2\pi\frac{i}{T_c}nT_s} \quad (36)$$

where $\grave{g}_i$ is the normalized Fourier series coefficients of the CCS, and N_g is the number of the CCS spectral line. Then the time-frequency cross correlation can be written as

$$R_c(\Delta\tau_p, w) = \frac{1}{N_s}\sum_{n=0}^{N_s-1}\sum_{i=-N_g}^{N_g} \grave{g}_i . e^{j2\pi\frac{i}{T_c}nT_s}\, e^{j2\pi\frac{w}{T_c}nT_s} = \sum_{i=-N_g}^{N_g} \grave{g}_i . \frac{1}{N_s}\sum_{n=0}^{N_s-1} e^{j2\pi\frac{i+w}{T_c}nT_s} \quad (37)$$

$$= \sum_{i=-N_g}^{N_g} \grave{g}_i . \operatorname{sinc}(2\pi\frac{i+w}{T_c}T_d) \cong \grave{g}_{-w} = \grave{g}_w$$

Form (37), it can be seen that, the time-frequency cross correlation $R_c(\Delta\tau_p, w)$ equals to the amplitude of the CCS spectral-line number w. Substitute (37) into (32), then the MSI signal power at the output of the correlator can be rewritten as

$$P_{MS}(\Delta\tau_p, w, \delta f_j) = K.P_j|\grave{g}_w|^2(\operatorname{sinc}(2\pi\delta f_j T_d))^2 \quad (38)$$

If $w = 0$ then

$$R_c(\Delta\tau_p, 0) = R_c(\Delta\tau_p) = \frac{1}{N_s}\sum_{n=0}^{N_s-1} P(n)\, C(n + \Delta\tau_p) = \grave{g}_0 \quad (39)$$

where $R_c(\Delta\tau_p)$ is the cross-correlation function for the C/A code. From (39), it can be seen that, the C/A code cross correlation function is equivalent to DC component of the CCS. In the previous section the correlator output power for MSI is shown in equation (38), substituting it in equation (4), the post-correlation C/No in the presence of MSI can be expressed as

$$C/N_o = \frac{P_G}{N_o + P_{MS}} = \frac{P_s K \left(1 - \frac{|\Delta\tau|}{T_a}\right)^2 (\mathrm{sinc}(\pi\Delta f_D T_d))^2}{N_o + T_d . K . P_j . |\grave{g}_w|^2 . (\mathrm{sinc}(\pi\delta f_j T_d))^2} \tag{40}$$

6. Analytical and Simulation Results:

This section introduces a simulation model (A computer based simulation using MATLAB has been employed in order to develop the simulation model) for the GPS receiver correlator. The GPS and interference signals at the correlator output and the post correlation carrier to noise ratio C/No are evaluated at different frequency shifts and integration time in order to investigate the interference impact.

The GPS signal power is chosen, as it is the minimum signal strength of a GPS signal reaching the user in an open sky environment $P_s = -158\text{dBw}$, a nominal received $C/N_o = 45\text{dB} - \text{Hz}$. The interference to received GPS signal power ratio $P_j/P_s = 20dB$. The coherent integration time take value from the range 1 to 20 msec, the non-coherent integration fixed to $K = 1$. For fair comparison the code phase estimate error is $\Delta\tau = 0$, the carrier phase estimate error is $\Delta f_D = 0Hz$, and the reference code in GPS receiver is PRN1.

Figure 3-a depicts the C/A code spectral-line amplitude for PRN1, and Figure 3-b depicts the C/No in presence of CWI when Δf_j is varying from -4 kHz to 4 kHz.

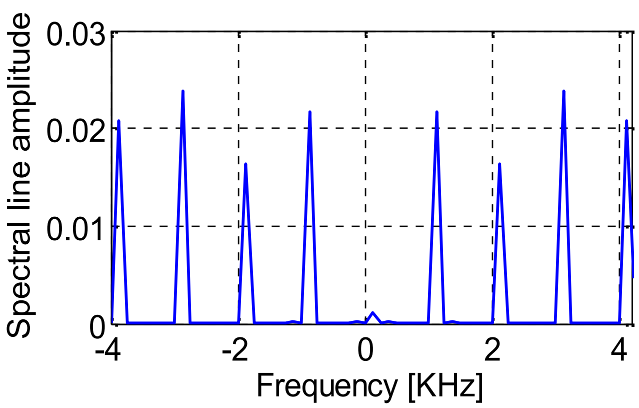

Figure 3-a: PRN1 spectral-line.

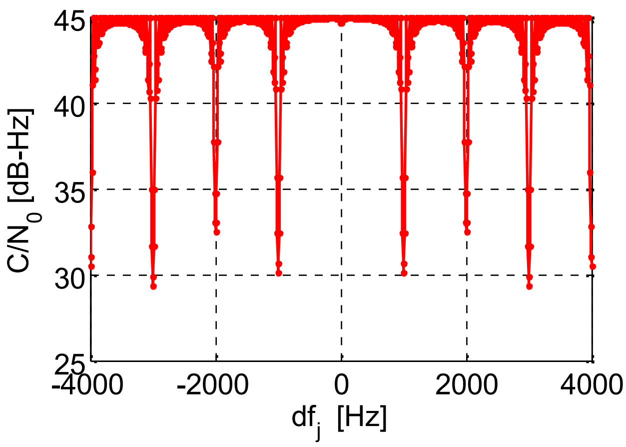

Figure 3-b: C/No in presence of CWI.

Figure 3: C/No in presence of CWI and C/A spectral-line.

From Figure 3, it can be seen that, when the CWI frequency varies, it will coincide with several C/A code spectral lines. Each of these lines has its own unique effect on the GPS receiver. When the CWI is far away from the received GPS signal carrier frequency, the CWI effect on GPS receiver will be greater than the case when the interference frequency equals to the received GPS signal frequency. This is done due to the fact that the C/A code spectral line number zero is too small in compared with the neighbor spectral lines as shown in Figure 3-a.

Figure 4 shows the post-correlation C/No is calculated in the presence of CWI, at different integration time T_d (1 to 20 msec), when the interference frequency error Δf_j is varying from 3.5kHz to 6.5kHz (with step 10 Hz) in the case of PRN1(C/A code for satellite1).

From Figure 4, it can be seen that, the width of the sinc trough (the CWI influence zone) around each spectral line is related to the integration time and equals to $\Delta f_u = \frac{2}{T_d}$. The CWI has a significant effect and it attenuates the receiver performance, when the difference between the interference frequency and the nearest GPS signal spectral lines δf_j is less than the reciprocal of the integration time. In other case, if the δf_j is more than the reciprocal of the integration time, the CWI needs very high power to effect the GPS receiver performance.

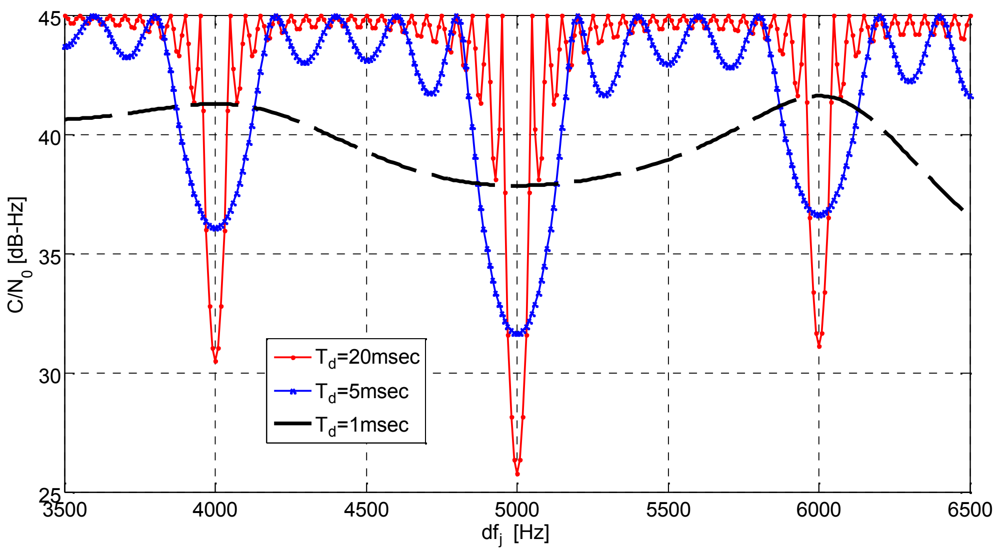

Figure 4: the post-correlation C/No in the presence of CWI at different T_d.

Figure 5 shows the correlator output power for the MSI using (38) at different integration time T_d (20msec, 10msec, 5msec), when the interference frequency error Δf_j is varying from 3.5kHz to 6.5kHz in the case $\Delta\tau_p = 5T_a$.

From Figure 5, it can be seen that, the MSI has a great correlator output power when the MSI spectral-lines coincide with the GPS reference signal spectral-lines. The graph is a

sinc function has width related to integration time and occurs every 1 kHz, and its amplitude is related to the amplitude of the CCS spectral-lines.
Figure 6 depicts the C/No in the presence of MSI when interference frequency error is varying from 3.5 to 6.5 kHz, and integration time take three value (20msec, 5msec, and 1msec).

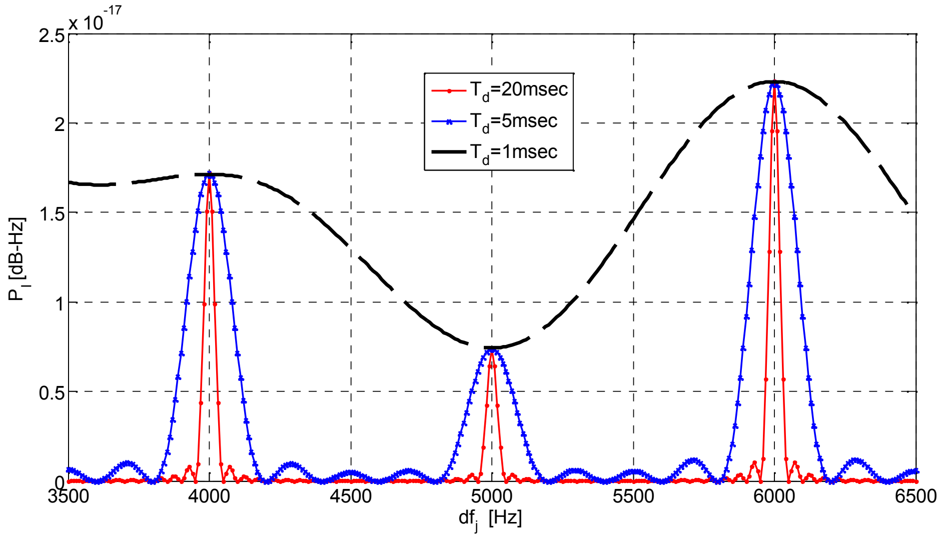

Figure 5: Correlator output power for MSI

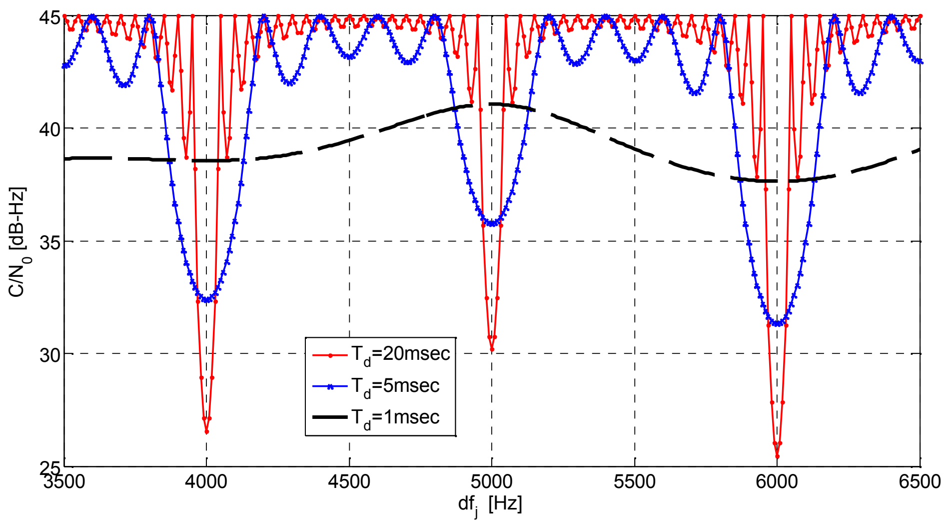

Figure 6: The C/No in the presence of MSI.

From Figure 6, it was shown that, when the MSI spectral-lines crosses the GPS signal spectral-lines there are drops in the C/No, the drop of a C/No value is dependent on the amplitude of the CCS spectral-line and integration time value.

The simulated results for correlator output power and C/No in presence of CWI are compared with the analytical results using the same parameters to verify the derived formulas.

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In Figure 7, the simulation and analytical correlator output power value for CWI is presented, The interference frequency error Δf_j is varying from 38 KHz to 46 KHz in the case of satellite-1 signal being acquired as shown in Figure 7-a, and from 259 kHz to 267 kHz in the case of a satellite-2 being acquired as shown in Figure 7-b.

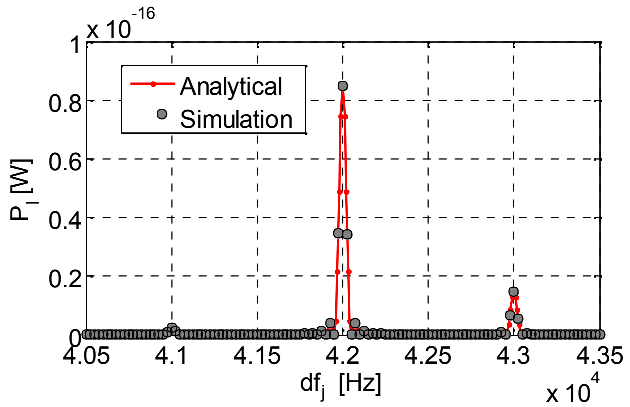

Figure 7-a: Correlator output power for CWI in the case of satellite-1.

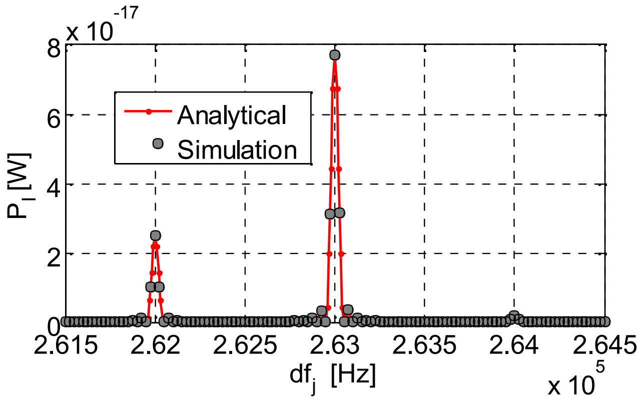

Figure 7-b: Correlator output power for CWI in the case of satellite-2.

Figure 7: Correlator output power for CWI.

From Figure 7, it can be seen that, the CWI correlator output power value depends on the amplitude of the C/A code spectral-line for C/A code. When the CWI is coincident with the greatest C/A code spectral-line (number 42 and 263 from the PRN1 and PRN2, respectively) it has highest correlator output power.

In Figure 8 depicts the comparison between simulation and analytical C/No value in presence of the CWI for PRN1 and PRN2.

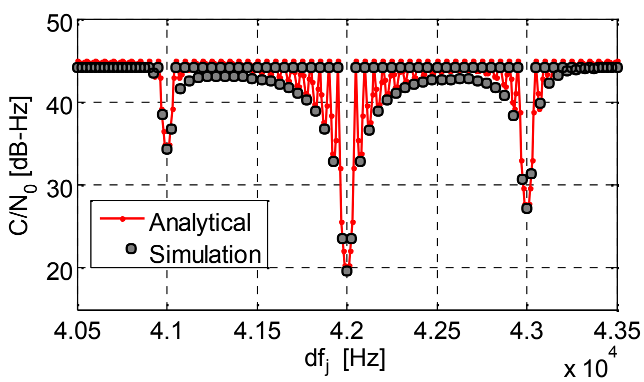

Figure 8-a: C/No in presence of CWI in the case of satellite-1.

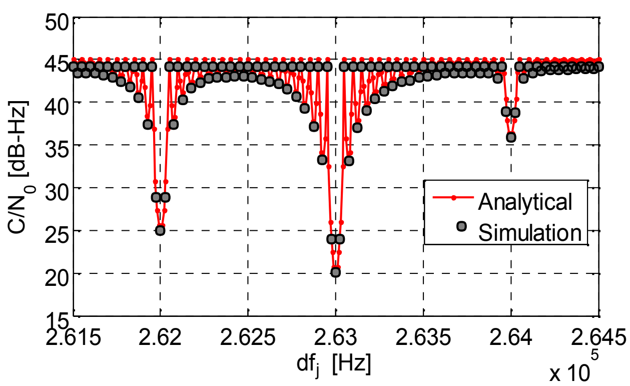

Figure 8-b: C/No in presence of CWI in the case of satellite-2.

Figure 8: C/No in presence of CWI.

From Figure 7, it can be seen that, as a CWI crosses the 1kHz spectral-lines of a GPS C/A code there are drops in the C/No. The GPS signals which transmitted by different satellites are affected differently by the same CWI. The CWI causes the greatest effect (minimum C/No value) on the GPS receiver, when the interference frequency is being at the specified offset with the GPS carrier frequency(42 KHz and 263 KHz from the PRN1 and PRN2, respectively).

Figure 7, and Figure 8 show that the simulation results for the CWI correlator output power and C/No, are close to the analytical result in (22) and (23), respectively.

The simulated results for correlator output power and C/No in presence of MSI are compared with the analytical results using the same parameters to verify the derived formulas. The Figure 9, depicts the correlator output power and C/No in presence of MSI when the interference frequency error Δf_j is varying from 224.5 KHz to 227.5, and correlator integration time $T_d = 20msec$.

It can be seen that, The MSI with $\Delta\tau_p = 5T_a$ and PRN2 code has highest effect on GPS receiver when the interference frequency error equal to 226 KHz (coincident with highest CCS spectral line when $\Delta\tau_p = 5T_a$).

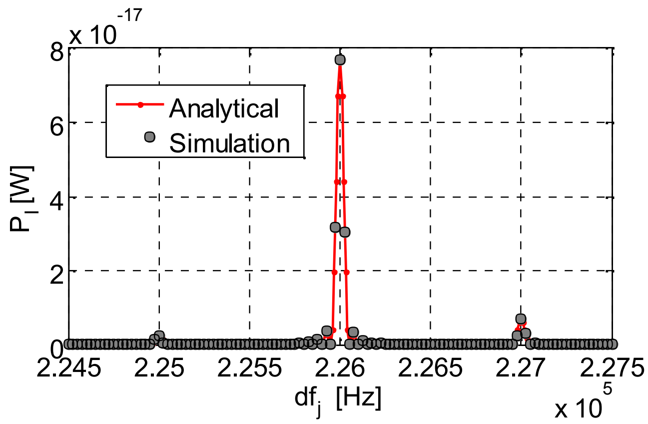

Figure 9-a: The MSI correlator output power

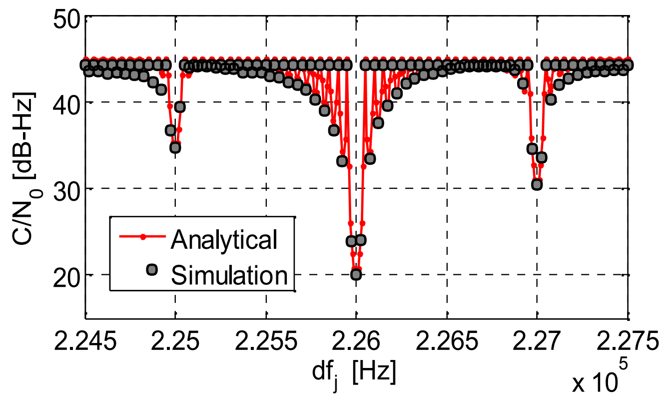

Figure 9-a: The C/No in presence of MSI

Figure 9: Correlator output power and C/No in presence of MSI.

Form Figure 9, it can be seen that, the agreement between the simulation and the analytical results supports the validity of the derived theoretical formulas (38)and (40).

7. Conclusions:

In this paper the post-correlation C/No and correlator output power for the GPS signal is investigated, in the presence of CWI, and MSI. It was shown that, when the CWI crosses the spectral-lines of a GPS signal there were drops in the C/No, the drop of the

C/No value was mainly dependent on the amplitude of the C/A spectral-line. When the CWI is far away from the received GPS signal carrier frequency, the CWI effect on GPS receiver will be greater than the case when the interference equals to the received GPS signal frequency.

The inverted Sinc function occurring around each trough in the C/No curve, and the Sinc function width were related to the integration time.

The drops in the C/No in the presence of MSI was dependent on the amplitude of the CCS spectral-line, and the drop occurred when the MSI spectral-lines cross the GPS signal spectral-lines.

When the GPS receiver coherent integration time increases and takes value more than 1ms, the GPS receiver will be more immune against CWI and MSI.

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