Stochastic Noise Characterization of Low Cost Inertial Sensors Using Allan Variance Technique

By

A.H. Hassaballa, A. M. Kamel, Y.Z. El-Halwagy

Abstract:

Inertial Navigation Systems (INS') are used as a primary mean of navigation in mostly all of the unmanned and autonomous systems. INS accuracy is categorized as grades in which the navigation grade is the most accurate and commercial grade is the least. The requirement of any used INS is to provide high accuracy information on the position, velocity, and attitude over a certain period of time. The problem of using low grade INS is that their accuracy degrades rapidly with time. To provide accurate estimation of navigation information, modeling of the sensors' noise components is required. The sensors' noise components are categorized in deterministic and stochastic parts. Deterministic noise such as bias and scale factor is easily removed in a process of laboratory calibration. Stochastic noise is the hardest part and needs special processes to be modeled and removed.

Allan Variance (AV) technique is a time domain method which can be used to characterize various types of stochastic noise terms appear in inertial sensor data by performing certain operations on an amount of data. In this paper, the relationship between different parameters which affect the operation of low cost Micro Electro Mechanical System (MEMS) inertial sensors such as sensor bandwidth and sampling rate is explored using Allan variance technique. Test results show that by carefully choosing internal inertial sensor settings, the sensor stochastic noise can be accurately modeled and hence, navigation processing is highly improved.

Keywords: Sensors, Allan variance, INS, Navigation.

1. Introduction

Navigation is the knowledge of how to determine position, velocity, and attitude of a moving object (e.g. a flight vehicle, a ship, a land vehicle, or a robots) during a certain period of time which also called navigation states. Inertial Navigation System (INS) is an integrated system in which a combination of measurement sensors measurements is used to determine all the navigation states using a signal processing which handle the model computations and integrations. An Inertial Measurement Unit (IMU) consists of three accelerometers combined together in an orthogonal arrangement and three gyroscopes arranged in the same manner as accelerometers. These sensors are jointly processed to obtain a full state estimation of the body[1]. The accuracy of obtaining the navigation states of the body depends on the grade of the IMU, such as tactical grade, navigation grade which their measurements can be used directly by strapdown inertial system algorithm due to their high accuracy but they are very expensive and low-cost grade (e.g. Micro elctro-mechanical systems(MEMS)) which have the advantage of small size, light weight but suffer from high noise that causes the INS to produce huge positioning errors in just few seconds[11]. If these errors are minimized then the navigation states drift of inertial system will be minimized.
IMU errors are classified into two categories; systematic and stochastic (random) errors. The calibration and characterization procedures became essential manner to improve the performance of MEMS accuracy. The calibration of a MEMS IMU is the process of comparing the instruments outputs with known reference information and the determination of the coefficients in the output equation, that agree to the reference information and used to compensate systematic errors, on the other hand the stochastic errors contains unpredictable random processes which appear on the output as a noise or a slow change of parameters in time these errors has to be modeled suing different techniques (e.g. Power Spectral Density (PSD), Allan Variance (AV), Autocorrelation Function (ACF)).

The rest of this paper is organized as follows:

1- Section (2) is discussing model of IMU error sources.
2- Section (3) is about how to compensate systematic errors.
3- Section (4) is presenting stochastic error sources identification.
4- Section (5) introduces experimental data for modeling MEMS IMU.
5- Section (6) is the summary and conclusions.

2. Mathematical models of IMU:

The performance of IMU can only be calculated by modeling the sensors comparing the IMU namely Gyroscopes and accelerometers which is presented in the next subsections.

2.1.Gyroscope measurement model:

Gyroscope is angular rate sensor that provide angular rate of the body and can be modeled by the following equation

$$\tilde{\omega}_{ib}^{b} = \omega_{ib}^{b} + b_g + S\omega_{ib}^{b} + N\omega_{ib}^{b} + \varepsilon_g \qquad (1)$$

where $\tilde{\omega}_{ib}^{b}$ is the measurement vector(deg/sec), ω_{ib}^{b} is the true angular rate velocity vector(deg/sec), $\mathbf{b_g}$ is the gyroscope instrument bias, **S** is the matrix representing the gyro scale factor, N is a matrix representing non-orthogonality of the gyro triad, and ε_g is a vector representing the gyro sensor noise (deg/s).

The matrices N_g and S_g are given as

$$N_g = \begin{pmatrix} 1 & \theta_{g,xy} & \theta_{g,xz} \\ \theta_{g,yx} & 1 & \theta_{g,yz} \\ \theta_{g,zx} & \theta_{g,zy} & 1 \end{pmatrix}$$

$$S_g = \begin{pmatrix} S_{g,x} & 0 & 0 \\ 0 & S_{g,y} & 0 \\ 0 & 0 & S_{g,z} \end{pmatrix}$$

where $\theta_{(\cdot),(\cdot)}$ are the small angles defining the misalignments between the different gyro axes and $S_{(\cdot),(\cdot)}$ the scale factors for the three gyros.

2.2.Accelerometer measurement model:

A linear accelerometer is an inertial sensor that measures the component of translational acceleration along its input axis. An output signal is produced from the force required to restore the proof mass to a null position relative to the case [1]. It has the same performance factors characterize the accuracy as the gyro.
Measurements of the specific force can be modeled by the following equation

$$\tilde{f}^b = f^b + b_a + S_1 f + S_2 f + N_a f + \delta g + \varepsilon_a \quad (2)$$

where $\tilde{f}^b$ is the accelerometer measurement vector(m\s ^{2}), f^b is the true specific force vector(m\s ^{2}), $\mathbf{b_a}$ is the accelerometer instrument bias vector(m\s^2), **S1** is the matrix of linear scale factor error, **S2** is the matrix of non-linear scale factor error, **N** is a matrix representing non-orthogonality of the accelerometer triad, δg is the deviation from the theoretical gravity value(m\s^2), and ε_g is a vector representing the accelerometer sensor noise (deg/s).

The matrices N_a and S_{a1} are given as

$$N_a = \begin{pmatrix} 1 & \theta_{a,xy} & \theta_{a,xz} \\ \theta_{a,yx} & 1 & \theta_{a,yz} \\ \theta_{a,zx} & \theta_{a,zy} & 1 \end{pmatrix}$$

$$S_{a1} = \begin{pmatrix} S_{a1,x} & 0 & 0 \\ 0 & S_{a1,y} & 0 \\ 0 & 0 & S_{a1,z} \end{pmatrix}$$

where $\theta_{(\cdot),(\cdot)}$ are the small angles defining the misalignments between the different accelerometer axes and $S_{(\cdot),(\cdot)}$ the scale factors for the three accelerometers.

2.3 Systematic (Deterministic) errors:

As shown in equation(1) and (2) the IMU errors can be defined as follows:[3] .

1-Bias offset:
It is defined as the output of the sensor when there is no input, This term often varies slowly with time so it is also called drift.

2-Scale factor:
It is the deviation of the input-output ratio from unity. The accelerometer output error due to scale factor is proportional to the true specific force along the sensitive axis, while the gyroscope is proportional to the true angular rate.

3-Non-orthogonality error:
It occur when any of the axes of the sensor triad deviated from mutual orthogonality during the manufacturing.

4-Misalignment Error:
It is the deviation between the sensitive axes of the inertial sensors and the orthogonal axes of the body frame due to mounting imperfection.

3.Calibration of MEMS IMU:

Calibration is used to compute the systematic errors of sensors in the lab using special procedure under certain environmental conditions using special standard devices, such as three-axis turntable to accurately determine all of the parameters.

3.1.Gyroscope calibration:

Because the low cost IMU gyroscope will not sense the angular rate of the earth $\boldsymbol{\omega_e}$ due to low accuracy of MEMS so The static position test calibration can't be used, another method called Angle rate test are utilized to compute biases, scale factor and non-orthogonalities of gyroscope.

The gyroscope to be calibrated mounted on a precision rate table which is rotated through a set of very accurately known angles rate. By comparing these known rotation with the estimates of the gyroscope reading, the various errors can be estimated. The table is rotated clockwise and anticlockwise through the same rates to compute gyroscope errors.

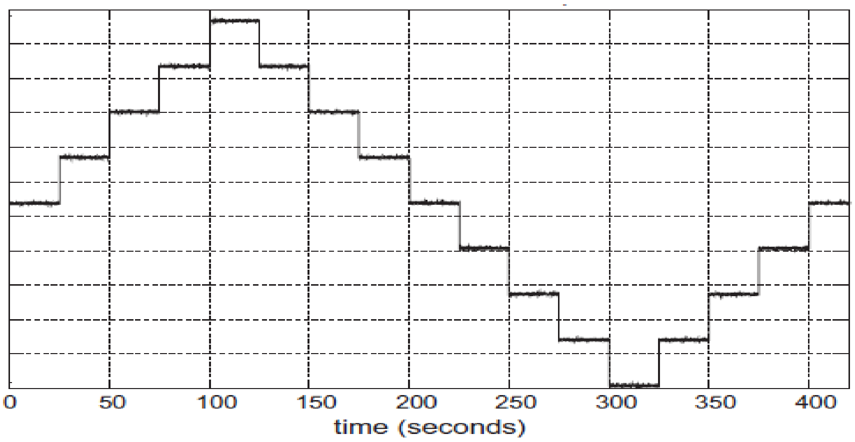

Figure 1: Gyro raw output data as response to a stepwise constant turn rate on the rate table.[4]

3.2.Accelerometer calibration:

The accelerometer to be calibrated is placed on a horizontal surface with facing up to sense gravity, after taking data for about 10-15 minute, the mean $\boldsymbol{f_{up}}$ is computed. Then place accelerometer facing down and collect data for same time, so specific force can be expressed as follow[3]

$$f_{up} = b_a + (1 + S_a)g \tag{3}$$

$$f_{down} = b_a - (1 + S_a)g \tag{4}$$

The bias b_a is computed by adding equations(3),(4)

$$b_a = \frac{f_{up} + f_{down}}{2} \tag{5}$$

and the scale factor S_a is obtained by subtracting equations (3),(4)

$$S_a = \frac{f_{up} - f_{down} - 2g}{2g} \tag{6}$$

where $\boldsymbol{g}$ is gravity of the earth.

The previous procedure is repeated of the three accelerometer sensors in IMU to obtain their individual bias and scale factors

4. Stochastic error sources identification:

Deleted: ¶

MEMS inertial sensors suffer from stochastic errors generated from the used low cost components that cause internal clock instability and sensitivity to temperature variations, so a stochastic modeling used to estimate probability distribution of potential outcomes by allowing fluctuation observed data in a certain time period, these distributions are derived from a large number of simulation which reflect the random variation in the input. In general manner white noise process is used for input signal, and by having the knowledge of the output only, the unknown model can be characterized. Many techniques are used to analyze the stochastic errors as mentioned earlier such as PSD, AV, and ACF.

4.1.Overview for PSD:

Deleted:

PSD is a powerful method used for analyzing data and stochastic modeling in frequency domain, it is defined to be the Fourier transform of its ACF $\Phi_{xx}(\tau)$.

$$\Phi_{xx}(f) = \int_{-\infty}^{\infty} \Phi_{xx}(\tau).e^{-j2\pi f \tau} d\tau, -\infty < f < \infty \tag{7}$$

4.2.Allan Variance Technique:

It is a time domain technique. There exists a unique relationship between AV and the PSD of the intrinsic random processes derived to be [6]

$$\sigma^2 = 4\int_{0}^{\infty} df . S_{\Omega}(f). \frac{\sin^4(\pi fT)}{(\pi fT)^2} \tag{8}$$

where $S_{\Omega}(f)$ is the PSD of the random process, the above equation show that the AV is proportional to the total power output of random process. So any physical meaning of the PSD can be obtained from the integral form AV, also from equation(8) it shows that the filter bandwidth depends on T. This suggest that different types of random processes can be examined by adjusting the MEMS filter bandwidth.

The AV is generally expressed in a log-log curve plot.

4.3.Error characteristic Using AV:

According to AV analysis, there are five error sources existing in inertial sensors as follows:

1. Angle/Velocity Random Walk(A/VRW):

It is a high frequency noise term that has a correlation time much shorter than the sample time and defined as additive white noise component on the sensor output. It represents by a line has a slope -1/2 and its value can be obtained directly by reading the slope line at τ=1.

2. Quantization Noise(Q):

It is strictly due to the digital nature of sensor output, obtained when sampling analog input signal using Analog to Digital (ADC). It represents by a line with slope= +1/2 and its value can be obtained directly by reading the slope line at τ==√3.

3. Flicker Noise (Bias Instability)(B):

It is mainly due to noise in electronics components and because of its low frequency nature it shows up as the bias fluctuation in the data, in MEMS IMU several factors cause bias to vary. It is represented in log-log plot by line has a slope=0

4. Angular Rate\Acceleration Random Walk(RRW):

It is random process of uncertain origin, it needs a long periods of collecting data in order to be able to observe it, usually output sensors affected by ambient temperature variation and spoils the RRW line on the AV, so it is recommended to run AV test in constant environmental condition. It represents by a line with slope= +1/2 and its value can be obtained directly by reading the slope line at τ=3.

5. Rate Ramp Noise(R):

It is more of deterministic error rather than stochastic noise, it shows due to a very slow periodic change of the sensor intensity persisting over a long period of time. It could be also due to a very small acceleration of the platform in the same direction and persisting over a long time period. It represents by a line with slope = +1 and its value can be obtained directly by reading the slope line at τ=√2.

6. Sinusoidal Noise:

A low frequency source could be the slow motion of the test platform due to periodic environmental changes.

4.4.Computing total effects of all processes:

A typical AV plot is shown in Figure 1. In practical measurements, different noise terms appear in different regions of τ .

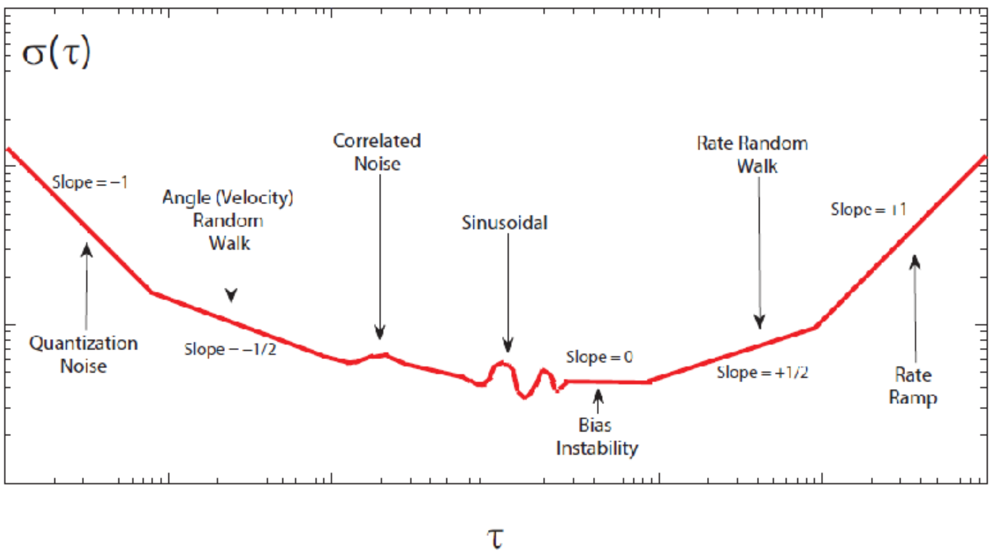

Figure 2: Sample plot of Allan variance analysis results (IEEE Std.952-1997)

Equation(9) shows the total AVs estimation

$$\sigma_{tot}^2(\tau)=\sigma_{ARW}^2(\tau)+\sigma_Q^2(\tau)+\sigma_B^2(\tau)+.......... \qquad (9)$$

Also the accuracy of the test design of AV modeling can be estimated, the accuracy depends on the number of independent clusters containing K data points within the total data set of N points as shown in equation(10).

$$\sigma = \frac{1}{\sqrt{2\left(\frac{N}{K} - 1\right)}} \tag{10}$$

Equation(10) shows that the estimation errors in the regions of short (τ) are small as the number of independent clusters in these regions is large and the estimation errors in the regions of long (τ) are large as the number of independent clusters in these regions is small

5.Experiments and Data Results:

In this paper, the MEMS IMU under test is from Invensense[5] and modeled as MPU-6050 shown in Figure 2, it has an embedded 3-axis MEMS accelerometer, a 3-axis MEMS gyros, and a digital motion processor hardware. Its specification shown in table (2).

Figure 3:MPU-6050 MEMS IMU

Table 1: Specification of MPU-6050

Sensor type	Parameter	Specification value	Units
Gyroscope	Full-Scale Range	± 250	°/s

	Sensitivity Scale Factor	131	LSB(°/s)
	Initial Zero Tolerance	± 20	°/s
	Total RMS Noise	0.05 at BW=100Hz	°/s-rms
	Rate Noise Spectral Density	0.005 at 10Hz	°/s/√H
Accelerometer	Full-Scale Range	± 2	G
	Sensitivity Scale Factor	16384	LSB/g
	Initial Calibration Tolerance	X,Y axes:±50 Z axis: ±80	Mg Mg
	Power Spectral Density	400 at 10Hz	μg/√Hz

Collecting static position data at different sampling intervals for 2 hours, average 600000 points of data was chosen analysis with AV. A vertically aligned accelerometer and gyroscope raw data is showed in Figure(3) and Figure (4) respectively before and after computing its deterministic errors.

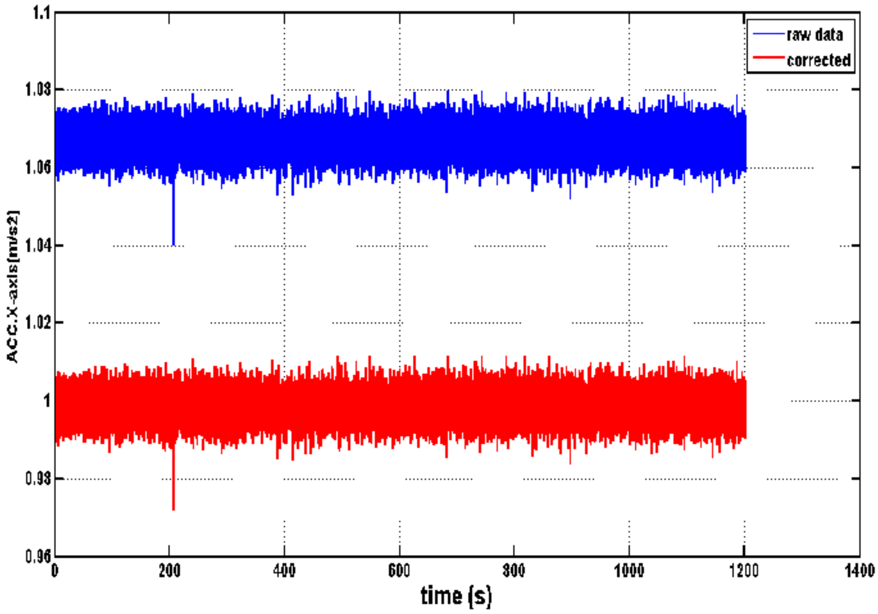

Figure 4:The accelerometer raw data before and after calibration

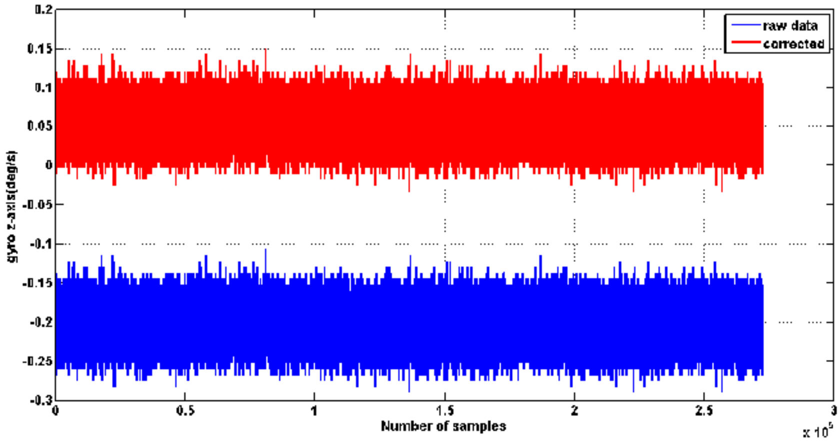

Figure 5: :original raw data of z-axis gyro in blue color and after correction in red color

A consistency check of the IMU is performed by running the same test under same conditions 3 times for different periods of time. The first test data is collected at a sample rate of 200Hz, bandwidth 42Hz, the second test collecting data at sample rate 50Hz, bandwidth 42Hz, and the third test collecting data at sample rate 50Hz, bandwidth 256Hz.

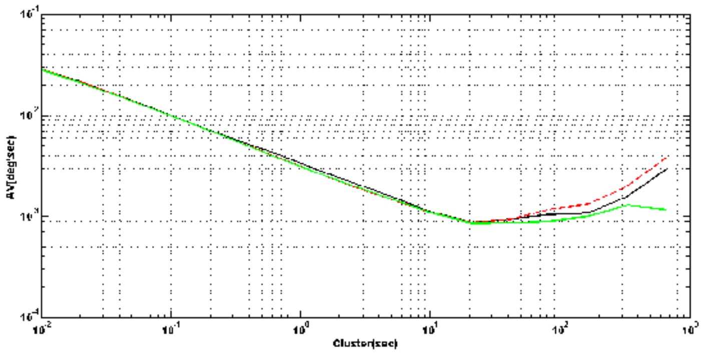

Figure 6: AV curve for z-gyro with sample rate 200Hz and BW=42Hz

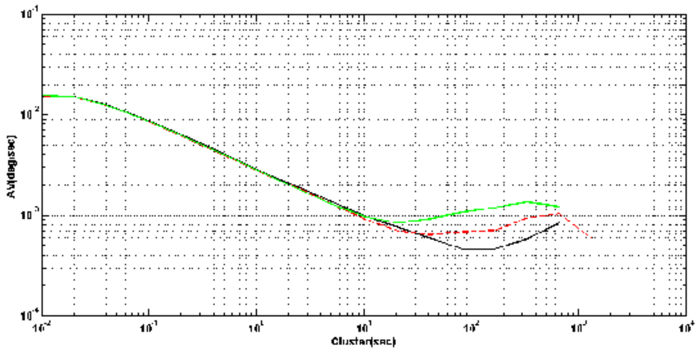

Figure 7: AV curve for z-gyro with sample rate 50Hz and BW=42Hz

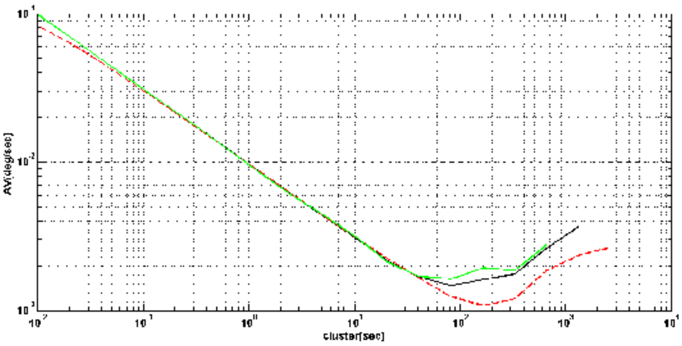

Figure 8:AV curve for z-gyro with sample rate 50Hz and BW=256Hz

As depicted from Figure(5)to Figure (7) ,for different mean times varies from tens to hundreds of seconds, the AV curve's slope of 0 -which represents bias instability- shows that several factors cause bias instability to vary in MEMS IMU units. It is usually estimated by averaging the sensor output when gyro is placed in horizontal position. It is found that the average value fluctuates with an increasing variance. Therefore, the more we wait, the more different values we obtain as bias.

5.1.Allan variance results:

after checking consistency and determine deterministic errors of MEMS gyros and accelerometers, the AV model ready to run at different sampling rate and different bandwidth to demonstrate the effect of sampling rate and sensor bandwidth. To determine a proper sampling rate and take advantage of full performance of MEMS sensor, compute the AV of the sensor output with relatively big sampling rate (200Hz) without digital filtering (BW=256Hz) and second run when internal digital filter was set to (BW=42Hz), and repeating test by down sampling rate to (50Hz) with both unfiltered data(BW=256) and filtered data (BW=42Hz)

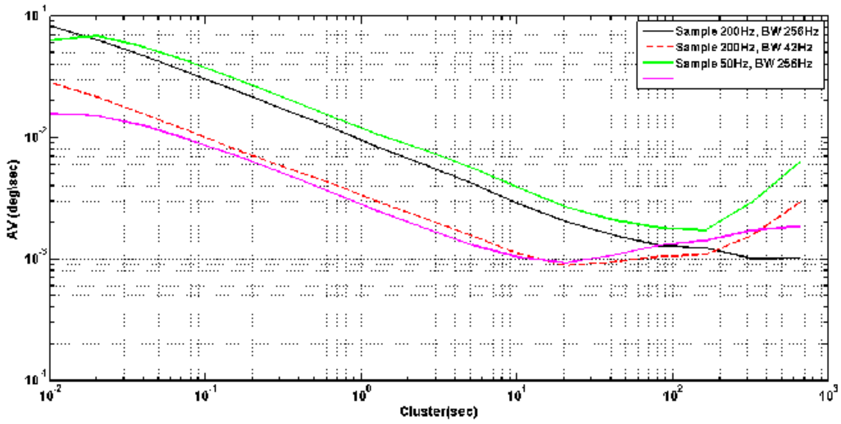

Figure 9:AV deviation curves for MPU-6050 ,z-gyro

Table 2: error coefficients for z-gyro

Sensor setting	rate =200Hz BW=256Hz	rate =200Hz BW=42Hz	rate =50Hz BW=256Hz	rate =50Hz BW=42Hz
Angle Random Walk(ARW)	0.00955	0.00337	0.01197	0.00283
Bias instability (B)	N/A	N/A	N/A	N/A
Quantization (Q)	N/A	N/A	N/A	N/A
Rate Random Walk (RRW)	N/A	N/A	N/A	N/A

The above results show that as long as signal sample rate approximately equal to the twice of the sensor bandwidth, the MEMS sensor will be at its full capacity in terms of its ARW (additive white noise) performance. On the other hand, trying to find a balance between gyro sample rate and its BW to get best value of ARW used in Kalman filter estimator design to get high accuracy navigation system with low cost and save micro-processor power.

$$Q = \sigma^2 \times INSperiod \quad (11)$$

Rate random walk

The most realistic solution for the determination of initial rate random walk power is to perform repeated calibration tests. Although performing repeated calibration tests under a variety of different environmental conditions takes too much time, unfortunately there is no other solution. In contrast to low cost MEMS units, the high-end inertial sensors usually do not contain any significant rate random walk error. Even if they contain any, it becomes significant only after a very-long continuous operation. Therefore, we almost never use an additional random walk components in the navigation Kalman fillers of high-end sensors. The affect of initial bias variations (due to the flicker noise) is accounted by slightly increasing the initial power of 1st order Markov processes which are used to model the short term flicker noise effects.

Ramp rate In low-cost MEMS sensors we deal with rate ramp as deterministic errors, so if it appears in the AV curve recalibration should be done for the sensor, be sure having a table power source, properly compensate for the temperature variation.

6.summary and conclusions:

Modeling MEMS inertial sensor is the most difficult step in the INS designs, the error characteristics suddenly change under different environments. Therefore, repeatability characteristics of low cost MEMS inertial sensors must be analyzed using a lot amount of calibration test. Also, our objective is to reach the best performance limit by adjusting the filter bandwidth and the sampling frequency in a proportional manner. If we use a very big bandwidth, we have to use a very big sampling rate to reach the limit which probably wastes micro-processor power. On the other hand, if we choose a very small bandwidth, then we may actually start filtering the motion itself rather than the noise. Therefore, we are trying to find a balance in between to reach the best value according to data sheet specification which used to compute process covariance matrix (Q) in Kalman filter estimator design to protect it from diverge and compensate position errors which yield to improve low- cost INS performance .

References

[1] Naser El-Sheimy, "Inertial Techniques and INS/DGPS Integration", Department of Geomatics Engineering, Calgary University-2007.

[2] R.K. Curey, M.E. Ash, L.O. Thielman and C.H. Barker , “Proposed IEEE inertial systems terminology standard and other inertial sensor standards”, Position Location and Navigation Symposium, pp. 83-90, Apr. 2004.

[3] Aboelmagd Noureldin, Tashfeen B. Karamat, Jacques Georgy "Fundamentals of Inertial Navigation, Satellite-based Positioning and their Integration" Springer

[4] Yigiter Yuksel, Naser El-Sheimy and Aboelmagd Noiueldin "Error Modeling and Characterization of Environmental Effects for Low Cost Inertial MEMS Units" 978-1-4244-5037-4/10,IEEE-2010

[5] InvenSense, MPU-6050 Product Specification . (http://invensense.com/mems/gyro/documents/PS-MPU-6000A-00v3.4.pdf.)

[6] IEEE STD952-1997(R2008). IEEE Standard Specification Format "Guide and Test Procedure for Single-Axis Interferometric Fiber Optic Gyros".

[7] IEEE STD647™-2006.IEEE Standard Specification Format "Guide and Test procedure for Single-Axis Laser Gyros".

[8] G.A. Aydemir, A. Saranlı,"Characterization and calibration of MEMS inertial sensors for state and parameter estimation applications" Journal of the International measurement Confederation- Measurement 45 (2012) pp.1210-1225.

[9] Claudia C. Meruane Naranjo "Analysis and Modeling of MEMS based Inertial Sensors", master thesis ,School of Electrical Engineering, XR-EE-SB 2008.

[10] Xin Zhang, Yong Li, Peter Mumford "Allan Variance Analysis on Error Characters of MEMS Inertial Sensors for an FPGA-based GPS/INS System" School of Surveying and Spatial Information Systems. University of New South Wales, Australia.

[11] "IMU & high performance inertial MEMS 2011, Complete review of inertial sensors market 2009-2015". Web: http://www.yole.fr.