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On The Study of Signal Bandwidth Limitations for Wideband DOA Estimation

By

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Abstract:

The paper presents the study of limitations of different techniques used for direction of arrival (DOA) estimation of uncorrelated and correlated wideband signals. The study was restricting the inter-sensor spacing of the array antenna to minimal wavelength of the applied wideband signal to avoid spatial ambiguity. The effect of the bandwidth on the estimation accuracy of wideband DOA is described for the case of two incident signals. First we analyzed two uncorrelated signals using MUSIC method. Second we suppose that the two signals are coherent using ML method. The study demonstrated that tradeoff between the signal bandwidth and DOA resolution of wideband signal. The paper suggests suggests reducing the array antenna elements as a solution for this tradeoff.

Keywords:

Multiple Signal Calcification (MUSIC), Direction of Arrival estimation (DOA), Maximum likelihood (ML), fast Fourier transform (FFT).

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1. Introduction:

Direction of Arrival estimation (DOA) is one of the most demanding problems which one has to solve for localizing and tracking multiple rapidly moving targets as in radar, mobile communication and in other areas.
A Linear antenna array consists of a set of antenna elements that are spatially distributed at known locations with reference to a common fixed point. By changing the phase and amplitude of the exciting currents in each of the antenna elements, it is possible to electronically scan the main beam and/or place nulls in any direction. The antenna elements can be arranged in various geometries, with the most common configurations along a line (linear array) [1].
This paper presents the study of the techniques that used for DOA estimation of uncorrelated and correlated wideband signals based on MUSIC and ML algorithms, and its Limitations on signal bandwidths.
The suggested method for wideband DOA estimation is based on dividing the frequency band into non-overlapping narrow bands (decomposed into several narrowband components via FFT or DFT), Then performing narrowband DOA estimation in each band. The DOA estimates for different bands are then combined to obtain the final DOA estimate by the MUSIC algorithm and Maximum Likelihood (ML).

2. Signal and Noise Models

Let's consider an M-sensor linear array in which the sensors are uniformly spaced. The distance between two imminent sensors is d .Assume the number of signal sources P is either known or can be estimated. The bandwidths w_L, w_H of the wideband sources. Then the m^{th}sensor output is

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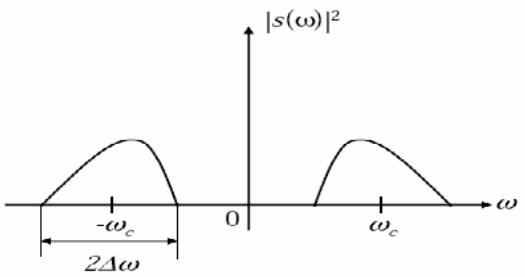


Figure (1): The energy spectrum of a real band pass signal

The bandwidth of signal is defined as the range of temporal frequency where the signal's

power is nonzero. Signals which have only one pass band are called band pass signal (See Figure(1)). According to the fractional bandwidth, which is the ratio of the bandwidth to the central frequency, the signal is either a narrowband signal or a wideband signal[2].

The frequency domain representation

$$x_m(t) = h_m(t) s * s(t-\tau_m) + n_m \qquad (1)$$

where h(t)is the impulse response of the m-the sensor and =1(spatial case)

The frequency domain representation

$$X_m(w) = H_m(w)[B(w-w_c) + B^*(w+w_c)]e^{-jw\tau_m} + N_m \qquad (2)$$

The Fourier transform of the demodulated signal is

$$X_m(w) = H_m(w+w_c)[B(w) + B^*(w-2w_c)]e^{-j(w-w_c)\tau_m} + N_m(w+w_c) \qquad (3)$$

The common received signal model for both narrowband and wideband sources.

$$X_m(w) = H_m(w_c)B(w)e^{-jw_c(1-w/w_c)\tau_m} + N_m(w+w_c) \qquad (4)$$

The input signal to the liner antenna array is described by

$$x_m(t) = \sum_{l=1}^{p} s_l(t - v_m \sin\theta_l) + n_m(t) \quad v_m = (m-1)d/c \qquad (5)$$

Narrowband Signals $B(w)\, is \quad 2\Delta w\, and\, \Delta w / w_c \quad << 1$

Wideband Signals $X_m(w) = H_m(w_c)B(w)e^{-jw_c\tau_m} + N_m(w+w_c)$ (6)

Wideband Signals It is supposed that there is a planer wave front impinging a linear array antenna, which has M, elements and an enter element space equal to Δ. Let ϕ be the direction of arrival of the planer wave front as shown Figure(2).

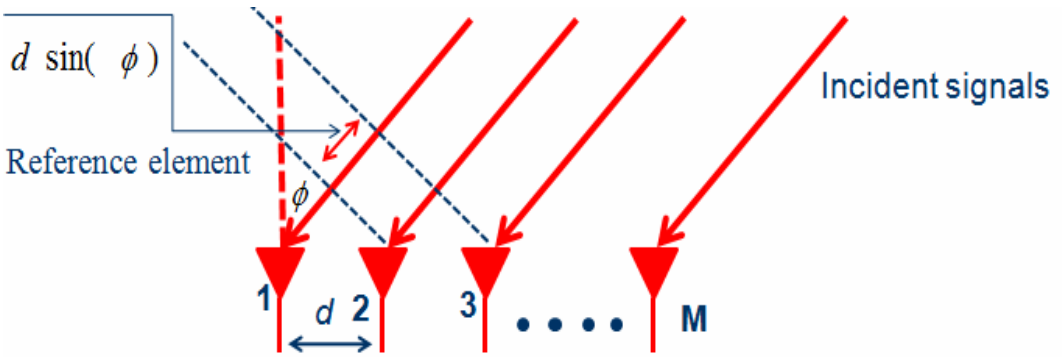


Figure (2): Geometry of linear array

measured with respect to the line of the elements. If the first element is taken as a reference point, the phase difference between the mth element and the first one is given by

$$\theta_{\mathrm{m}} = \frac{2\pi}{\lambda_{\mathrm{w}}} \Delta_{\mathrm{m}} \sin(\phi)$$

Where λ_w is the operating wavelength.
The signal and noise at the array elements as a vector x(t)

$$x(t) = A(\phi)s(t) + n(t) \qquad (7)$$

Multipath model $\quad x(t) = A_c(\phi)s_c(t) + A_u(\phi)s_u(t) + n(t)$

Correlation matrix $\quad R_{xx} = E\left[x(t)x^H(t)\right] = A(\phi)E\left[s(t)s^H(t)\right]A^H(\phi) + E\left[n(t)n^H(t)\right]$

$$= A(\phi)R_s A^H(\phi) + \sigma_n^{\ 2} I \qquad (8)$$

From Wideband signals to Narrowband signals

$$x_m(t) = \sum_{l=1}^{p} s_l(t - v_m \sin\theta_l) + n_m(t) \qquad v_m = (m-1)d/c \qquad (9)$$

$s_l(t)$ is the lth signal source
$n_l(t)$ is noise at the mth sensor
ϕ_l is the DOA to be estimated

The DFT of the mth sensor output (the received signal) of all elements, is divided into K frequency by DFT as

$$\begin{aligned} X(w_k) &= A(w_k,\phi)S(w_k) + N(w_k), k = 1,.....,K \\ X(w_k) &= [X_1(w_k) \quad X_2(w_k)..... \quad X_M(w_k)]^T \\ S(w_k) &= [S_1(w_k) \quad S_1(w_k)..... \quad S_P(w_k)]^T \qquad w_L < w_k > w_H \\ N(w_k) &= [N_1(w_k) \quad N_1(w_k)..... \quad N_M(w_k)]^T \end{aligned} \qquad (10)$$

Then the correlation matrix described as $\quad R(w_k) = E\left[X(w_k)X^H(w_k)\right]$

$$R_{xx} = E\left[x(t)x^H(t)\right] = A(\phi)E\left[s(w_k)s^H(w_k)\right]A^H(\phi) + E\left[n(w_k)n^H(w_k)\right]$$

$$= A(\phi)R_{ss}A^H(\phi) + \sigma_{nn}^{\ 2} I \qquad R_{xx} \quad decompose \quad V(w_k) and U^H(w_k)$$

$V(w_k) \quad U^H(w_k)$ are the signal and noise subspace of frequency respectively.

The equation (10) represents the model of the input data and noise that is most

commonly used in the next sections.

3. Direction Of Arrival (DOA) Estimation for narrow band signal

3.1 MUSIC Method

The word MUSIC is an abbreviation of (Multiple Signal Classification). This method exploits the eigen-structure of the signals covariance matrix[3].
we can tell that there are two subspaces related to the eigenvectors resulted by the covariance matrix R_x:

1- Noise subspace: consists of the M-D eigenvectors corresponding to the smallest eigenvalues as $E_n=[e_D\ e_{D+1}\ldots e_{M-1}]$.
2- Signal subspace: consists of the D eigenvectors corresponding to the largest eigenvalues as $E_s=[e_0\ e_1\ldots e_{D-1}]$.

From the orthogonality property of the steering vectors corresponding to the signal components and the noise subspace vectors we can write that

$$a^H(\phi)E_n E_n^H a(\phi) = 0 \qquad (11)$$

$$\begin{bmatrix} a^H(\phi_0)e_m \\ a^H(\phi_1)e_m \\ M \\ a^H(\phi_{D-1})e_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ M \\ 0 \end{bmatrix}$$

Then the DOAs of the multiple incident signals can be estimated by locating the peaks of the MUSIC spectrum given by

$$f_{music}(\phi) = \frac{1}{a^H(\phi)E_n E_n^H a(\phi)} \qquad (12)$$

As noticed from the denominator, the orthogonality between $a(\phi)$ and E_n will make it minimum, and hence will increase $f_{music}(\phi)$. Hence the $\hat{D}$ largest peaks of the MUSIC spectrum correspond to the DOAs of the signals impinging on the array.

3.2 Maximum Likelihood (ML) Method

Maximum likelihood is a good method compared with subspace methods from point of its ability to estimate DOAs in low SNR condition and at small number of signal samples [4].

The received signal matrix is expressed as

$$X = A(\phi)S + N \qquad (13)$$

Base on the assumption made about the nature of noise, and assuming that the signals are unknown deterministic sequences rather than random processes, the joint probability density function (PDF) of the output data is expressed as.

$$F(X) = \prod_{k=0}^{N-1} \frac{1}{\pi \det[\sigma^2 I]} \exp\left(\frac{\left| x(k) - A(\phi)s(k) \right|^2}{-\sigma^2} \right) \qquad (14)$$

From equation (4.73), the ML estimate of the DOAs can be obtained by maximizing the Log-Likelihood function

$$\hat{\phi} = \max J(\phi) = \max \sum_{k=0}^{N-1} \left| P_{A(\phi)} x(k) \right|^2 \qquad (15)$$

Where $P_{A(\phi)} = A(\phi)(A^H(\phi)A(\phi))^{-1} A^H(\phi)$

$$\hat{\phi} = \max J(\phi) = \max \ trace[P_{A(\phi)} \hat{R}_x] \qquad (16)$$

So $J(\phi)$ is the ML spatial spectrum. The maximization of (16) is a nonlinear multidimensional problem, which is computationally very intensive. So alternatively, we will plot the ML spatial spectrum and then find its peaks.

4. Direction Of Arrival (DOA) Estimation for Wideband signal

For many applications, wideband signals are exploited for localization. A wideband signal is any signal whose energy is distributed over a bandwidth that is large in comparison to the signal's center frequency. There are two main approaches for wideband beam forming based on time-domain processing and frequency- domain processing, for signals with large bandwidths, the frequency-domain approach offers computational advantage over the time-domain approach. A structure of a frequency-domain processing beamformer is shown in Figure (3). In this beamformer, wideband signals from each element are transformed into frequency domain using the fast Fourier transform (FFT)as shown Figure (3), and each frequency bin is processed by a narrowband processor [5].

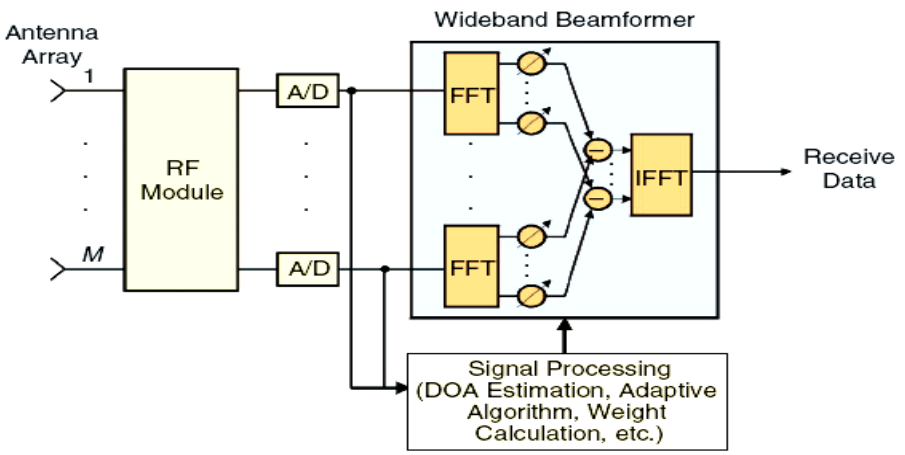


Figure.(3): Adaptive array configuration for receiving wideband signals

4.1 Incoherent wideband signal (MUSIC Algorithm)

The music algorithm is derived under the narrowband signal assumption. it cannot be directly applied to the wideband. on approach is to split the wideband signal into many sub-bands in frequency such that each sub-band can handled by the MUSIC (I-MUSIC) algorithm the incoherent MUSIC spectrum is defined below,

$$P_{MUSIC}(\phi) = \frac{1}{\frac{1}{P_{MUSIC}(f_1,\phi)} + \frac{1}{P_{MUSIC}(f_2,\phi)} + \frac{1}{P_{MUSIC}(f_3,\phi)} + \ldots + \frac{1}{P_{MUSIC}(f_k,\phi)}}$$

$$\hat{\phi} = \arg\max P_{MUSIC}(f,\phi) \qquad (17)$$

4.1.1 Simulation

A coded simulation has been carried out to evaluate this method. In the simulation, we assume that three uncorrelated wide band signals impinging a linear array of number of elements (M=11) at angles of arrivals -30° , 0° and 40°. The three signals are assumed to have equal signal to noise ratio (SNR=10 dB), the number of snapshots taken from the array is N=64. The bandwidth of the signal is taken as half of the carrier frequency (f_r). This bandwidth will be divided into 64 sub bands. Then using MUSIC method and applying Monte-Carlo simulation with 50 trails, the detected signals computed using MUSIC algorithm against angles are shown in Figure (4). It is obvious that the detected peaks indicating the DOAs of the three signals which typically agree with the incident signals.

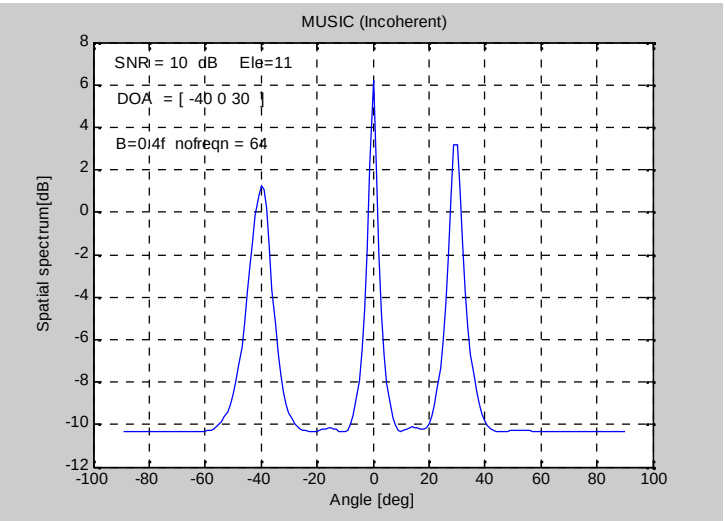


Figure (4): The spatial spectrum of the MUSIC estimator (Uncorrelated signal)

4.2 Coherent wideband signal (Maximum Likelihood Algorithm)

While the subspace-based methods presented in the subspace method as (MUSIC) are computationally attractive, they do not always yield sufficient accuracy. In particular, for scenarios involving highly correlated (or even coherent) signals, the performance of subspace-based methods may be insufficient. An alternative is to exploit the underlying data model, leading to so-called parametric array processing methods. The price to pay for this increased efficiency and robustness is that the algorithms typically required multidimensional search to find the estimates. The most well known and frequently used model-based approach in signal processing is the maximum likelihood (ML) technique. The DOA can be estimated using (18).

$$P_{ML}(\phi) = \frac{1}{\frac{1}{P_{ML}(f_1,\phi)} + \frac{1}{P_{ML}(f_2,\phi)} + \frac{1}{P_{ML}(f_3,\phi)} + \ldots\ldots + \frac{1}{P_{ML}(f_k,\phi)}}$$

$$\hat{\phi} = \arg\max P_{ML}(f,\phi) \qquad (18)$$

4.2.1 Simulation

A simulation program has been carried out to verify the capability of the ML algorithm estimator. In the simulation we considered the same assumptions that we have taken in the previous simulation program of the MUSIC except that the two signals completely coherent wide band s1(t) and s2 (t) respectively coming from the array from -30 and 40 degrees .

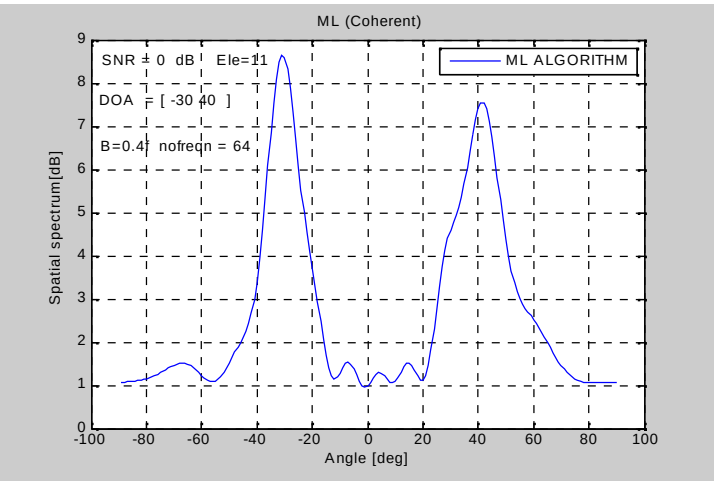


Figure (5): The spatial spectrum of the ML estimator (Correlated signal)

The performance of the ML algorithm is in first case, the incident signals are assumed to be SNR 0 dB. The detected signal computed using the ML algorithm against angles is presented in Figure (5). It is clear that the ML algorithm has successfully determined the correction DOA of the incident signals. It is worth to mention that the success in the DOA estimation has been satisfied for lower SNR compared to the case previously introduced in Figure (4) of uncorrelated signals.

5. Effect of the Bandwidth on the Estimation Accuracy

Here we used the MUSIC and ML algorithms to study the effect of the bandwidth on the accuracy of the estimation. In the simulation program we take the same assumptions that we have taken before, except that we frequently change the bandwidth of the signals in the rang (0.3f-1.1f) by a step size 0.1f , and each time we measure the DOAs. The frequency contents of the signal is considered to occupy the region

$$f_l<f<f_h \qquad (19)$$

With $f_l=f_{rf} - B/2$ and $f_h=f_{rf} + B/2$

Where B is bandwidth of the signal. The frequencies used in the division of the bandwidth into sub-bands in the simulation are given by

$$f=f_l,f_l+\Delta f,\ f_l+2\Delta f,\ldots\ldots\ldots\ f_h-\Delta f,\ f_h \qquad (20)$$

Where

$$\Delta f=(f_h - f_l)/N \qquad (21)$$

With N being the number of time samples.

5.1 Simulation

A simulation program has been carried out to verify the capability of the MUSIC and ML algorithms estimator. In the simulation we considered the same assumptions that we have taken in the simulation program of the MUSIC and the parameter of incident signal are:

1- $f_{low} = 2.4000e+009$ $f_{high} = 3.6000e+009$ *- band=0.45*carrf*

carrfgn = 3.0000e+009 - space betwwen element = 0.5 wavelength -

SNR=10dB.

Accordingly, the detected signals were computed for the different bandwidth employing both MUSIC and ML algorithms. The detected signals in each case are illustrated in Figure (6).

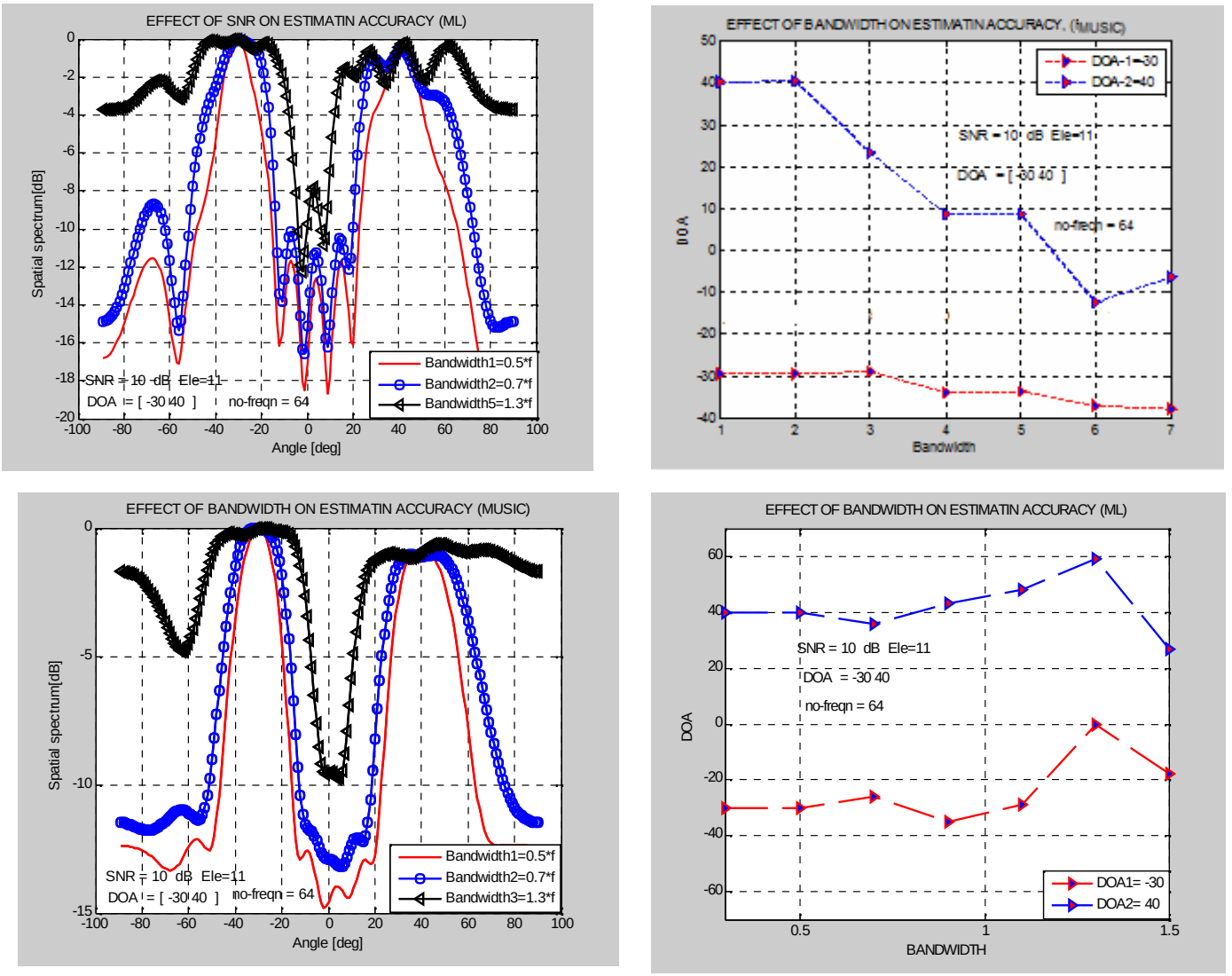


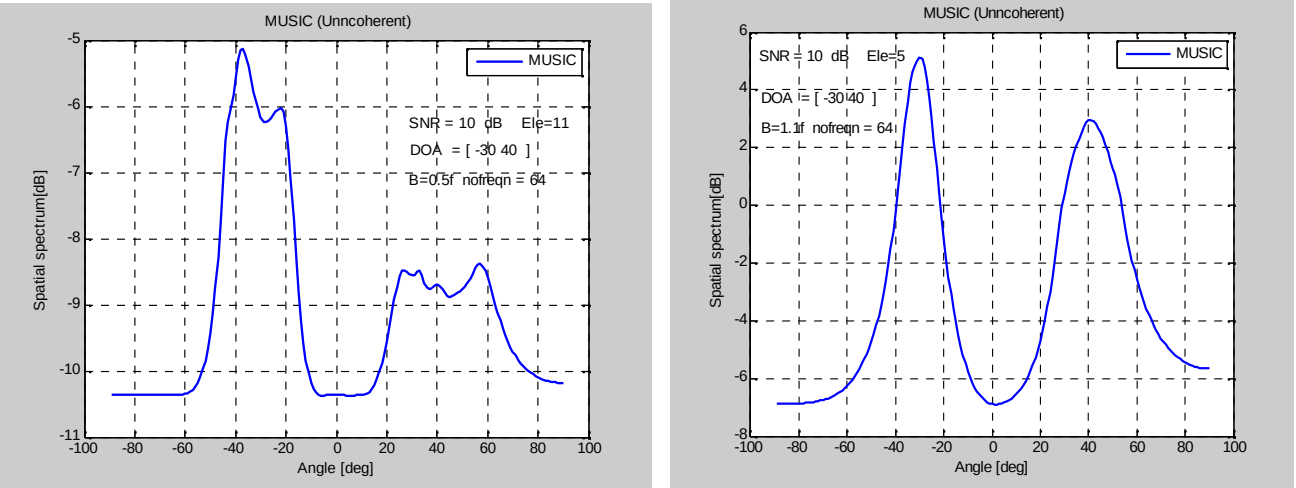
Figure (6): Effect of the Bandwidth on the Estimation Accuracy (MUSIC and ML)

As shown in Figure (6), it is obvious in the figure; the accuracy gets worse as the bandwidth increases in both algorithms. Quantities by values can be investigated in Figure (6) by observing the detected signal level against incident angles shown in Figure (6), left side, or the detected DOA angles plotted against the frequency in Figure (6), right side.

In our work in this paper, one suggested solution to overcome this problem was by changing the number of elements. Through very parametric simulation, we have found that the solution is possible by decreasing the number of elements. For the sake of paper length, we could not include this study in this manuscript. Hence, we have suggested decreasing the number from 11 elements to only 5 elements and we have investigated the MUSIC algorithm for the following wideband signal parameters.

2- $f_{low} = 1.3500e+009$ $f_{high} = 4.6500e+009$ *- band=1.1* carrf*
carrfng = 3.0000e+009 - space betwwen element = 0.5 wavelength -SNR=10 dB

For the case of using 11 elements and band is 1.1* carrf the results is shown in Figure (7-a). It is obvious that the DOA fails in determining the second signal DOA at -30 degree and pour resolution at 40 degree, on the other hand, by decreasing the number of elements to five, the DOA estimation is shown in Figure (7-b). It is clear that the DOA has significant enhancements.



(a) (b)

Figure (7): Effect of the Bandwidth on the Estimation Accuracy (MUSIC and ML)

5. Conclusions

In this paper the performance Of MUSIC and ML have been investigated using an

adaptive array antenna. The results have shown that the MUSIC method can resolve uncorrelated sources only. The ML method can resolve uncorrelated sources as well as coherent sources. For wide band signals the estimator accuracy degraded as the bandwidth increases. Finally, we have demonstrated that by decreeing the number of elements, the DOA has been solved with limited resolution.

6. References

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