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Parameter Estimation of Composite Continuous Wave Signals in the Presence of Non-Gaussian noise

By

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Abstract:

An algorithm for estimation of the amplitudes and phases of composite Continuous Wave (CW) signals is considered. The signals are contaminated with non-Gaussian noise. The considered noise model is the most commonly exist in communication applications. The developed algorithm for estimating the amplitudes and phases of composite CW signals is based on the Expectation Maximization (EM) algorithm. The most feature of the developed algorithm is that it reduces the complicated multi-parameters optimization required when using the ML approach. The complexity of the algorithm is essentially unaffected by increasing the number of composite signals. Simulation experiment is performed to illustrate the performance of the developed algorithm.

Keywords:

Spectral Analysis, Expectation Maximization.

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1. Introduction:

The problem of parameters estimation of multiple sinusoids in noise has a great attention in the signal processing literature. Some methods were published to solve this problem. Most of these methods are based on the Maximum Likelihood (ML) approach [1-5]. The basic idea to all of the estimation schemes in above references consists of obtaining sub-optimal initial estimates of the parameters and then refining them through maximization of the likelihood function. The sub-optimal initial estimate method and the maximization step are varying from reference to reference. Some methods use Prony's least square estimator which employes the least square to fit an auto regressive model to the noisy sinusoidal data [6]. James introduced another approach based on the Extended Kalman Filter (EKF) [7]. This approach is considered an approximate conditional mean estimator. This estimator is found to be asymptotically efficient for sufficiently signal to noise ratio.

All the above-described methods do not consider the presence of non-Gaussian noise. They only consider white Gaussian noise. In this paper, we consider additive non-Gaussian noise and we develop an algorithm to estimate amplitudes and phases of composite CW signals based on the Expectation Maximization (EM) approach. The paper is organized as follows. In section II, the problem statement and noise model are presented. In section III, the mathematical formulation of the developed algorithm is introduced in some details. Section IV demonstrates the simulation results of the developed algorithm. Finally, the paper is concluded in section V.

2. Problem Statement and Noise Model

The received observation, $r(t)$, of composite CW signals in the presence of non-Gaussian noise c(t) can be expressed as:

$$r(t) = \sum_{i=1}^{I} a_i \cos\,(\omega_i\, t + \theta_i) + c(t) \qquad (1)$$

where a_i; (i=1, 2, …I), is the amplitude of the i-th signals, ω_i is the angular frequency of the i-th signal, θ_i is the phase of the i-th signal, I is the number of the CW signals which is assumed to be known and c(t) is a non-Gaussian process, modeling the noise. The problem can be stated as follows. Given the observation r(t), which consists of a known number of composite signals plus an additive white non-Gaussian noise, it is required to estimate the amplitude parameters a_i and the phase parameters θ_i of each CW signal. In the following we discuss the considered noise model in some details. The noise model is a product of a real, non-negative, wide sense stationary process, s(t), times a Gaussian process g(t), independent of s(t). Thus, c(t) is given by $c(t)=s(t)\,g(t)$. This model is physically consistent with some important disturbance phenomena, such as atmospheric noise, scattering from the sea surface and, more generally, from rough surfaces in remote sensing applications. If the observation time is short with respect to the coherence time of the modulating process, then s(t) can be approximated by a random constant s, called auxiliary variate and consequently the non-Gaussian noise becomes c(t)=s g(t). Then we can say that c(t) is a Gaussian process with stochastic mean square value or equivalently it is conditionally Gaussian random process given s. The process c(t) can be real or complex according to g(t). In this paper, c(t) is considered real because we deal with real signals. Direct application of the probability law to the process c(t) leads to the following expression for the probability density function (pdf) $f_c(.)$ of the noise c(t) [8]

$$f_c(C)=\int_o^{\infty}\frac{1}{\sqrt{2\pi\ s^2\,\sigma^2}}\exp\left(\frac{-C^2}{2\sigma^2\,s^2}\right)\ f_s(S)\ ds \qquad (2)$$

where $f_s(S)$ is the marginal pdf of s and σ^2 is the variance of g(t). The noise representation described above applies if and only if a pdf $f_s(S)$ satisfying the Fredholm type-I integral equation (given by (2)) exists. Most distributions of relevant practical interest satisfying equation (2); among them the Generalized Laplace, the Generalized Cauchy, the Generalized Gamma and the Contaminated Normal [8]. It is noted that the above distributions depend on at least two parameters.

3. Mathematical Formulation

The received observation, $r(t)$, of the composite CW signals using the above non-Gaussian noise model can be expressed as:

$$r(t)=\sum_{i=1}^{I}a_i\cos\left(\omega_i\,t+\theta_i\right)+s\,g(t) \qquad (3)$$

The likelihood function of the observation $\Delta[r(t);a_i,\theta_i]$ can be written as

$$\Delta[r(t);a_i,\theta_i]=\int_0^{\infty}\frac{1}{\sqrt{2\pi s^2\,N_0}}\exp\left(\frac{-\int_T\left|r(t)-\sum_{i=1}^{I}a_i\ \cos\left(\omega_i+\theta_i\right)\right|^2 dt}{2s^2\,N_o}\right)f_s(S)\,ds \qquad (4)$$

where N_o is the height of the power spectral density of the Gaussian component of c(t). The Maximum likelihood approach can be used to estimate a_i and θ_i; (i =1, 2,…,I) by maximizing (4) with respect to these parameters. Since the integral in the numerator is a monotonic decreasing function of the argument of the exponential and $f_s(S)$ is non-negative function and independent of a_i and θ_i, then maximizing (4) is equivalent to minimize the argument in the numerator with respect to a_i and θ_i. That is:

$$\min_{a_i,\,\theta_i} \int_T \left| r(t) - \sum_{i=1}^{I} a_i \cos(\omega_i\, t + \theta_i) \right| dt \qquad (5)$$

This minimization step is a complicated multi-parameter optimization problem (2I-dimensional search minimization problem) that tends to be computationally complex and time consuming. The Expectation maximization (EM) algorithm is used to solve this problem and reduce the involved computations. An overview of the EM algorithm is found in [9]. Since (4) is conditionally Gaussian given s with conditional variance $s^2 N_o$, then $\Delta[r(t); a_i, \theta_i]$ can be written as

$$\Delta[r(t); a_i, \theta_i] = \int_0^\infty f_{r/s}(R/a_i, \theta_i, S)\; f_s(S)\, ds \qquad (6)$$

where $f_{r/s}(R/a_i, \theta_i, S)$ is the conditional pdf of the observation given a_i, θ_i; (i =1, 2,…,I) and S. As stated before, since $f_s(S)$ is a non-negative function and independent of a_i and θ_i, then maximizing (6) is equivalent to maximize the conditional pdf of R given S=s, that is maximizing $f_{r/s=s}(R/a_i, \theta_i, S=s)$ with respect to a_i and θ_i. This problem is solved using the EM algorithm as follows.

Define x as the complete data which has a pdf $f_x(.)$. In our problem the choice of the complete data x(t) is given by decomposing r(t) into I signals, that is:

$$x(t) = [\, x_1(t) \;\; x_2(t) \; ... \; x_I(t)]^T \qquad (7)$$

where $x_i(t) = a_i \cos(\omega_i\, t + \theta_i) + c_i(t)$ and $c_i(t)$ are chosen to be uncorrelated zero-mean non-Gaussian noise satisfying $\sum_{i=1}^{I} c_i(t) = c(t)$. It is clear that $x_i(t)$ is conditionally Gaussian given S=s. The relation between the complete data and the incomplete (observed) data is

$$r(t) = \sum_{i=1}^{I} x_i(t) = Q\; x(t) \qquad (8)$$

where Q consists of I terms and is given by $Q = [1,1,...,1]$. Since $x_i(t)$, (i=1, 2, …, I) are statistically independent, the log-likelihood of $x(t)$ is given by:

$$\log f_{x/s=s}(x/S=s,a_i,\theta_i)=\sum_{i=1}^{I}\log f_{x/s=s}(x_i/S=s,\, a_i,\theta_i) \qquad (9)$$

where the log-likelihood function of $x_i(t)$ is given by:

$$\log f_{x_i/s=s}(x_i/S=s,a_i,\theta_i)=-\frac{1}{2}\log\left(2\pi s^2 N_i\right)-\frac{1}{2s^2 N_i}\int_T\left|r(t)-a_i\cos(\omega_i t+\theta_i)\right|^2 dt \quad (10)$$

Where Ni = qi .No and qi 's are arbitrary non-negative real-valued scalars satisfying $\sum_{i=1}^{I} q_i = 1$. Substituting of (10) into (9) and carrying out the conditional expectations required by the EM algorithm, the following algorithm is obtained:

E-step For i=1, 2,..., I compute

$$\hat{x}_i^{(n)}(t)=\hat{a}_i^{(n)}\cos(\omega_i t+\hat{\theta}_i^{(n)})+q_i\left[r(t)-\sum_{l=1}^{I}\hat{a}_i^{(n)}\cos\left(\omega_i t+\hat{\theta}_i^{(n)}\right)\right]$$

M-step For i=1, 2, …, I

$$\min_{A,\Theta}\int_T|\hat{x}_i^{(n)}(t)-A\cos(\omega_i t+\Theta)|^2 \rightarrow \hat{a}_i^{(n+1)},\hat{\theta}_i^{(n+1)}$$

where n denotes the iteration number. The two-parameter minimization required in the M-step can be carried out in two steps as follow [9]. Define

$$L_i^{(n)}(\theta)=\int_T \hat{x}_i^{(n)}(t)\cos(\omega_i t+\theta)\,dt$$

Then we obtain $\hat{\theta}_i^{(n+1)}$ from the following maximization step:

$$\max_{\theta}\ |L_i^{(n)}(\theta)|$$

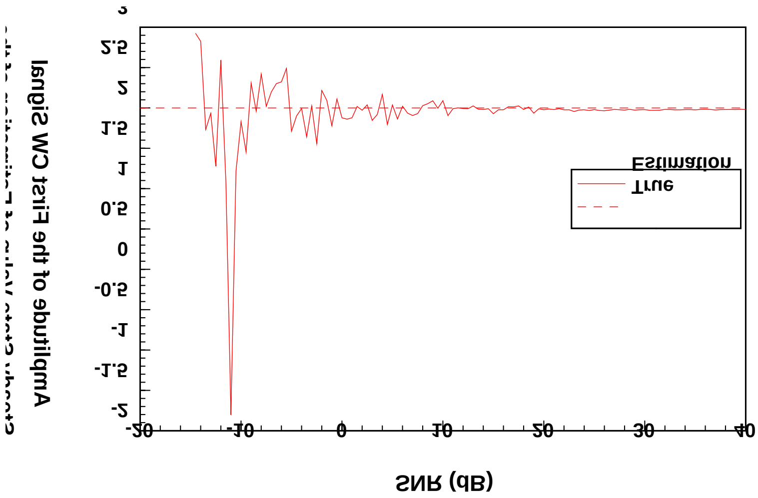
then $\hat{a}_i^{(n+1)}$ can be obtained from the following step:

$$\hat{a}_i^{(n+1)} = \frac{L_i^{(n)}(\hat{\theta}_i^{(n+1)})}{E_i}$$

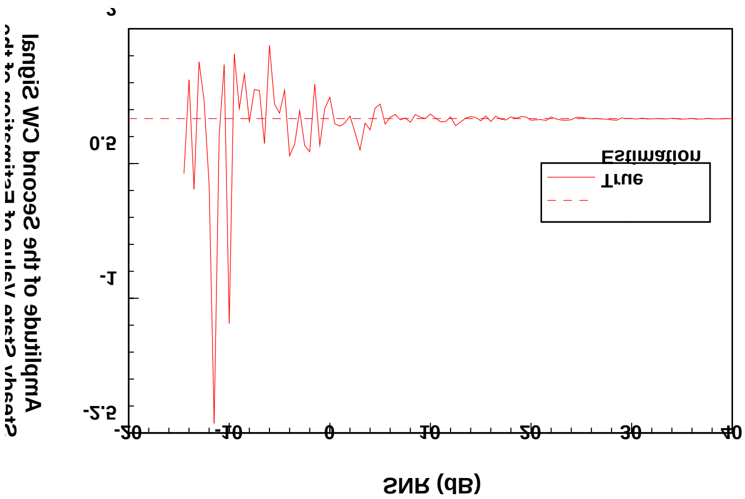
Where E_i is given by $E_i = \int_T \cos(\omega_i t)\, dt$. This algorithm can be implemented as follows. The signal $L_i^{(n)}(\theta)$ can be generated by passing $\hat{x}_i^{(n)}(t)$ through a filter matched to $\cos(\omega_i t)$ and then search for the hightest peaks of the I matched filters. The algorithm is illustrated in Fig. 1. The algorithm decreases iteratively the objective function in (5) without ever going through the indicated multi-parameter optimization. The complexity of the algorithm is essentially unaffecred by the number of signals. As the number of signals increases, only the number of matched filters increases in parallel and each matched filter is maximized separately. Then using this algorithm, the 2I-dimensional search is reduced to a I dimensional search.

4 Simulation and Results:

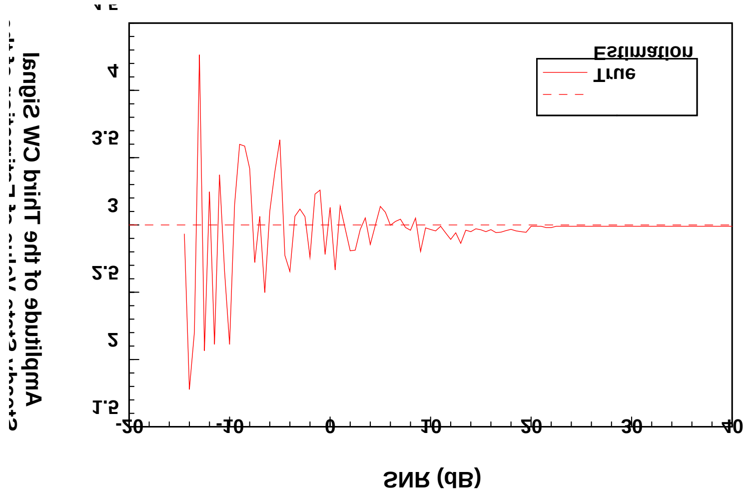
In this section, the performance of the estimation algorithm is evaluated. Four CW signals are generated with amplitudes 2, 1, 3, and 0.5 and phases $\pi/5$, $\pi/6$, $\pi/4$, and $\pi/2$. The generated CW signals are added to the non-Gaussian noise. The probability distribution function of the random parameter s in the non-Gaussian noise model is chosen as a Gamma distribution. The estimation algorithm is applied to estimate the amplitudes and the phases of the four signals. The steady state value of estimation of the amplitudes and the phases versus the SNR is shown in Fig.2 and Fig.3 respectively. These figures show that as the SNR increases, the algorithm converges to the true values. Finally, we can conclude that the developed algorithm can simultaneously estimate the amplitudes and phases of composite CW signals in the presence of non-Gaussian noise.


Amplitude of the First CW Signal
SNR (dB)
True
Estimation

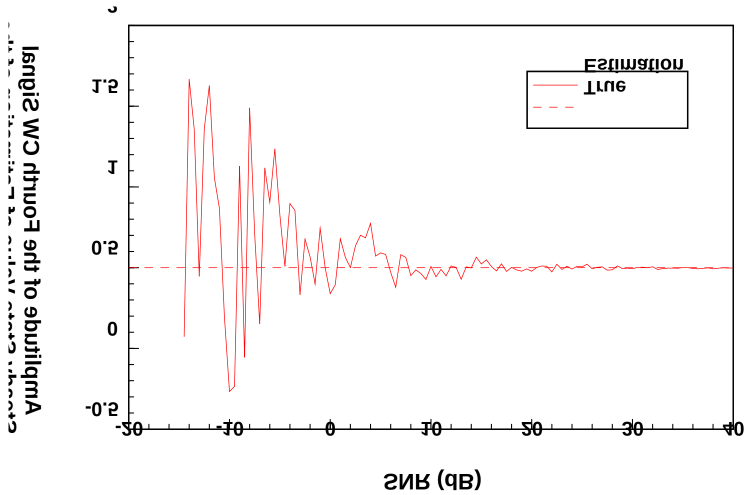
(a)


Amplitude of the Second CW Signal
SNR (dB)
True
Estimation

(b)

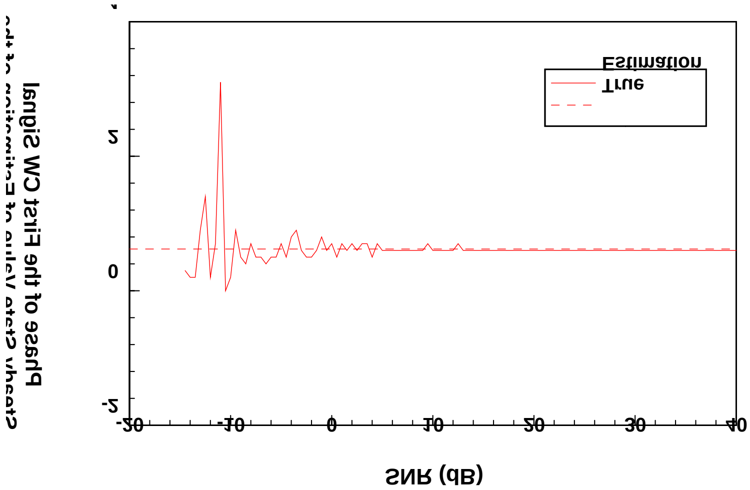


(c)

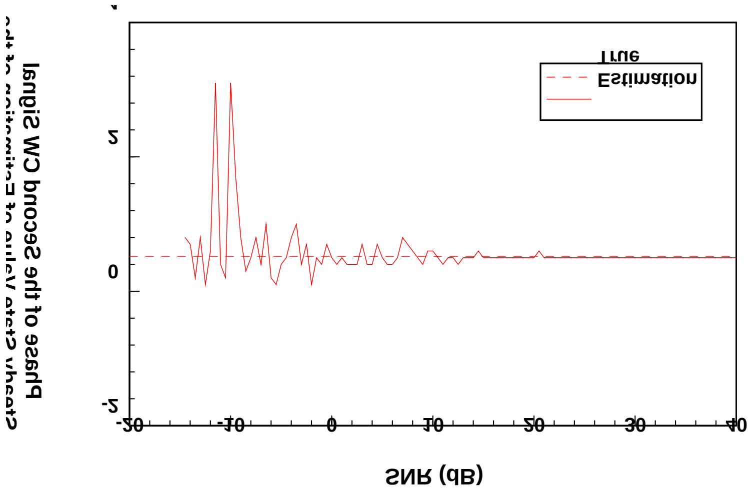


(d)

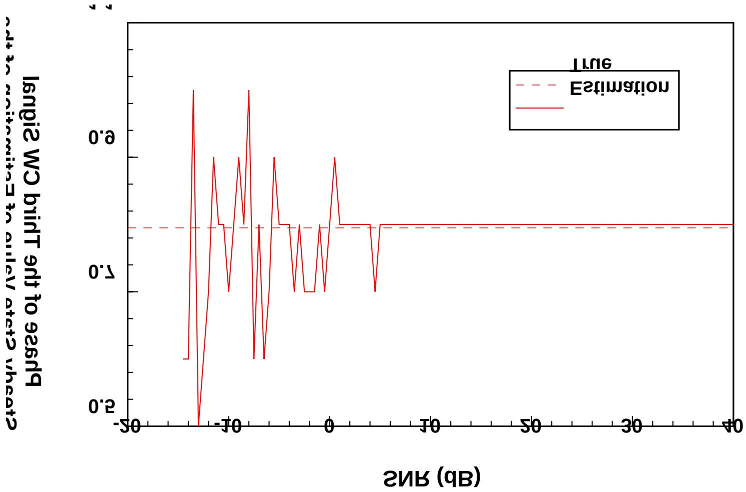
Fig. 2: Steady State Value of Estimation of the Amplitudes


Phase of the First CW Signal
Estimation
True
SNR (dB)

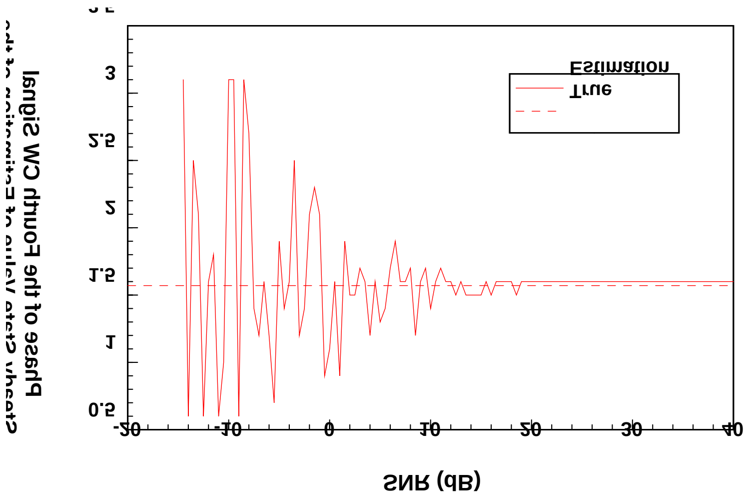
(a)


Phase of the Second CW Signal
True
Estimation
SNR (dB)

(b)



(c)



(d)

Fig. 3 : Steady State Value of Estimation of the Phases

Conclusion:

An algorithm for estimation of the amplitudes and phases of composite Continuous Wave (CW) signals has been developed. The signals are contaminated with non-Gaussian noise. The developed algorithm is based on the Expectation Maximization (EM) algorithm and reduces the complicated multi-parameters optimization required by the ML approach. The complexity of the algorithm is essentially unaffected by increasing the number of composite signals. Simulation experiment illustrates that the estimation algorithm can simultaneously estimate the amplitudes and phases of composite CW signals in the presence of non-Gaussian noise.

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