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Performance of CDMA IRIDIUM Low Earth Orbit Satellite Systems with Coding

By

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Abstract:

The effect of The effect of block, convolutional and Turbo coding on the probability of error and the capacity are investigated for CDMA IRIDIUM Low Earth Orbit (LEO) Satellite systems. The model employed assumes a contaminated Gaussian traffic model. The conventional Gaussian distribution can be considered as a special case.

Keywords:

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1. Introduction:

Low earth orbit satellite communication systems are one of the most appropriate systems to offer personal communications (PC) [1-3]. They can also provide additional advantages for the global communication networks, e.g., small propagation delay and loss, and high elevation angle in high latitude [4].
One of the most recent candidates for establishing the multiple access in LEO satellite systems is Code Division Multiple Access (CDMA). CDMA has higher capacity than TDMA and FDMA if voice activity and frequency reuse by spatial isolation are employed [5]. The non-uniform distribution of the traffic is a normal feature of our globe. However there are only few studies on the effect of this non-uniformity of the traffic on the capacity of LEO systems. Capacity analysis of LEO satellite communication systems with traffic non-uniformity was considered in [6]. In this later paper, the traffic model was assumed to have a Gaussian probability density function with variance ω2, which represents the traffic non-uniformity. The model represents the case of an isolated city. Due to the relative large area coverage of a LEO satellite, more than one inhabited area can exist in the coverage area of three consecutive satellites. In this case the Gaussian distribution will not be a good choice to represent the traffic non-uniformity. A (rather) more suitable distribution for a lot of practical cases is the Gaussian mixtures, which is considered in this paper.
This paper suggests a general traffic model that can resemble specific areas in the globe. It also discusses the effects of traffic non-uniformity and coding on the performance of the LEO satellite communication system employing CDMA scheme. In section 2 we define the suggested traffic model. Section 3 reviews expressions for the probability of error for block, convolutional, and turbo coding techniques. Section 4 discusses how the capacity measurement is obtained. Effects on the probability of error and capacity are introduced in Section 5. Section 6 displays the numerical results. Finally, conclusions are included in Section 7.

2. The Contaminated Gaussian Traffic Model:

In LEO satellite systems, the satellites are organized on a multiple orbit configuration. In the proposed system for example the 66 satellites are organized in 6 orbits, each with 11 satellites. For the sake of simplicity and to make the effect of traffic non-uniformity more clear we will consider the simple 2- dimensional model with a single orbit as in [6].
In this model shown in Fig.1 an arc represents an area on the earth [7]. The figure shows the coverage and interference areas of each satellite. The coverage area is specified by the minimum elevation angle (θmin). The interference area of a satellite is specified by

the final line of sight of that satellite. An area covered by two satellites will be denoted by “double coverage area”. B_i represents the position of the i[th] satellite measured from the center of the earth. To analyze the effect of traffic non-uniformity, we define a general traffic distribution as the sum of Gaussian distributions with different parameters

$$P(\alpha) = A \sum_{i=-M}^{M} \frac{\varepsilon_i}{\omega_i} \exp\left(\frac{-(\alpha-\mu_i)^2}{2\omega_i^2} \right), \qquad |\alpha| \le \pi, \quad 0 \le \varepsilon_i \le 1 \tag{1}$$

Where α is the angular distance of any user from the origin measured by the angle at the center of the earth in radians, μ_i is the center of the i[th] populated area, ω_i is the nonuniformity parameter of the i[th] populated area and ε_i is the weight of the i[th] populated area relative to the total traffic load. The number of populated areas will depend on “M” and the values of εi as will be shown later. The constant A is given by:

$$A = B \left[\int_{-3\pi/N_s}^{3\pi/N_s} \sum_{i=-M}^{M} \frac{\varepsilon_i}{\omega_i} \exp\left[\frac{-(\alpha-\mu_i)^2}{2\omega_i^2} \right] d\alpha \right]^{-1} \tag{2}$$

Where B is the total traffic load for the three satellites between $-3\pi/N_s < \alpha < 3\pi/N_s$, N_s is the number of satellites in one orbit. For the sake of simplicity we will assume that $\omega_i=\omega$ for all i.

Here we examine how the degree of the traffic nonuniformity affects the performance of the system. In the case where one satellite, say, the i[th] one, is above the traffic peak, the signal to interference ratio (SIR) at the i[th] satellite becomes[7]

$$SIR_i = 0.5 \left[\int_0^{\pi/N_s} P(\alpha) d\alpha + \int_{\pi/N_s}^{\beta_I} P(\alpha) I_{i+1}^2(\alpha) I_i^{-2}(\alpha) d\alpha \right]^{-1} \tag{3}$$

Next, we shall consider three cases of the distribution of the users.

<u>2.1. The conventional Gaussian distribution:</u>

In this case P(α) is given by:

$$P(\alpha) = \frac{A}{\omega} \exp(-\alpha^2 / 2\omega^2), \qquad |\alpha| \le \pi \tag{4}$$

Which is the same distribution assumed in [6, 7].

<u>2.2. The bimodal conventional Gaussian distribution:</u>

In this case, the conventional contaminated Gaussian distribution is given by:

$$P(\alpha) = \frac{A}{\omega} \left[\varepsilon \exp\left[\frac{-(\alpha-\mu)^2}{2\omega^2} \right] + (1-\varepsilon) \exp\left[\frac{-(\alpha+\mu)^2}{2\omega^2} \right] \right],$$

$$|\alpha| \le \pi, \qquad 0 \le \varepsilon \le 1 \tag{5}$$

when μ=0 and for any value of ε in the above equation, equation (4) is obtained.

2.3. The trimodal conventional Gaussian distribution:

In this case $P(\alpha)$ is given by:

$$P(\alpha) = \frac{A}{\varpi}\left[\varepsilon_1 \exp\left[\frac{-(\alpha-\mu)^2}{2\varpi^2}\right] + \varepsilon_2 \exp\left[\frac{-\alpha^2}{2\varpi^2}\right] + \varepsilon_3 \exp\left[\frac{-(\alpha+\mu)^2}{2\varpi^2}\right]\right],$$

$$|\alpha| \le \pi, \qquad 0 \le \varepsilon \le 1 \tag{6}$$

Equations (5) and (6) are suggested distributions which can represent in the real world the case of two or three populated areas respectively.

3. Probability of Error for Different Coding Algorithms:

In this section we will review the expressions for the probability of error without coding and when using block, convolutional and turbo coding techniques.

3.1. Without coding:

The bit error rate for BPSK is given by [8]

$$P_e = Q\left(\sqrt{\frac{2E_b}{(N+I)}}\right) \tag{7}$$

Where $E_b/(N+I)$ is the signal-to-noise+interference ratio per bit.

3.2. Block code:

A block code (B) is a mapping of k input binary symbols into n output binary symbols where n>k. Cosequenttlly, the block coder is memoryless device (each bit in the code word is independent of the previous bits). The codes are denoted by (n , k), where the code rate R is defined by R = k/n

A maximum of t errors per codeword will be corrected by the decoder since the block code is assumed to be able to correct up to t errors. A word or block of k message bits will be incorrectly decoded when more than t errors occur in the n bit codeword. Thus [9], P_w = P((t+1) or more errors occur in an n bit codeword)

$$P_w \approx \binom{n}{t+1}(q_c)^{t+1}(1-q_c)^{n-t-1} \tag{8}$$

where P_w is the probability of incorrectly decoding a word (block) of message bits in the coded system. qc is the channel bit error probability for the coded system and is given by:

$$q_c = Q\left(\sqrt{\frac{2E_b}{(N+I)}\frac{k}{n}}\right) \tag{9-a}$$

That is the equivalent signal to noise + interference per bit is given by:

$$\left(\frac{E_b}{(N+I)}\right)_B = \left(\frac{E_b}{(N+I)}\right)\left(\frac{k}{n}\right) \tag{9-b}$$

Hence the average message bit error rate is given by:

$$P_e = \frac{(t+1)}{n} P_w \tag{10}$$

3.3. Convolutional coding

Convolutional codes (C) are powerful coding schemes for wireless mobile communication systems. We can define the constraint length as the number of shift register in the convolutional encoder.

The probability of bit error when using Viterbi decoding is given by [8]:

$$P_e \le \frac{1}{k} \sum_{d=d_{\min}}^{\infty} a(d) P_2(d) \tag{11}$$

where $P_2(d)$ represents the pair wise error probability, a(d) is the number of bit errors in all adversaries at distance d from the correct codeword/sequence and d_{min} is the minimum distance of the code. The probability of selecting the incorrect path for d even and hard decision decoding is given by [8]:

$$P_2(d) = \frac{1}{2}\binom{d}{d/2}\left[P(1-P)\right]^{d/2} + \sum_{k=1+d/2}^{d}\binom{d}{k} P^k (1-P)^{d-k} \tag{12}$$

where P is the probability of a bit error without coding. and for d is odd is given by:

$$P_2(d) = \sum_{k=(1+d)/2}^{d}\binom{d}{k} P^k (1-P)^{d-k} \tag{13}$$

3.4. Turbo coding:

A turbo encoder consists of two parallel concatenated convolutional encoders called coonstituent codes (Recursive Systematic encoders (RSC)) separated by an interleaver, with an optional puncturing mechanism [10].

The probability of error for Turbo coding (T) is bounded by:

$$P_e \approx \max_{w \ge 2}\left[\frac{w n_w}{k} Q\left(\sqrt{\frac{2 R d_{w,\min}^{TC} E_b}{(N+I)}}\right)\right] \tag{14}$$

where n_w and $d_{w,\min}^{TC}$ are functions of the particular interleaver employed, w is the weight of the data word, $d_{w,\min}^{TC}$ is the minimum weight turbo codeword produced by w. R is the code rate, and k is each block of incoming data bits.

4. Capacity Measurement:

An important performance measure in LEO satellite is the system capacity. To calculate the capacity, we first determine the ratio of the bit energy-to-noise plus interference density for the i^{th} satellite [6]:

$$(E_b/(N+I))_i = \frac{BW/R_b}{(I_i/S_i)+(\eta/S_i)} \qquad (15)$$

where the numerator is the ratio of the total bandwidth (BW), to the information bit rate (Rb), and the denominator is the total interference-to-signal ratio plus the ratio of background noise (η), to signal. The total interference reaching the ith satellite Ii is proportional to the total traffic load B. Therefore the maximum amount of traffic that the system can support for a given condition can be denoted as:

$$B_i = \frac{I_i}{[I_i]_{B=1}} \qquad (16)$$

where $[I_i]_{B=1}$ means the interference I_i calculated at B=1. Solving (15) for I_i and substituting it in (16) we have

$$B_i = \left[\frac{BW/R_b}{(E_b/(N+I))} - \frac{\eta}{S_i}\right].\frac{S_i}{[I_i]_{B=1}} \qquad (17)$$

From this equation, we can derive the maximum traffic B_i (capacity) for a certain $E_b/(N+I)$ on the i^{th} satellite for a given BW, R_b, η, and S_i. The capacity is dependent on $E_b/(N+I)$.

5. Evaluation of Probability of Error and the Capacity for LEO Satellites when Coding is Employed:

In the following, we are going to show how the probability of error and the capacity are evaluated using different coding techniques.

5.1. Block code:

For a certain ω we calculate SIR from equation (3), $E_b/(N+I)$from equation (15) and the probability of error from equation (7) without coding. When block coding is employed equation (9-a) is substituted in equation (8) and then equation (8) in equation (10) to get the probability of error. To obtain the capacity, substitute equation (9-b) in equation (17).

5.2. Convolutional coding:

Taking the probability of error without coding for a certain ω and substituting in equation (12) or equation (13) and equation (11). Then solving equation (11) using numerical solution we get the corresponding $E_b/(N+I)$. Finally by substituting this value in equation (17) we get the required capacity.

5.3. Turbo coding:

Taking the probability of error without coding for a certain ω and substituting in equation (14). Then solving equation (14) using numerical solution we get the corresponding $E_b/(N+I)$. Finally by substituting this value in equation (17) we get the required capacity.

6. Numerical Results:

The probability of error and the capacity are calculated for the three consecutive satellites S1, S2 and S3 for the case of one, two and three populated areas respectively assuming that all received satellites power are equal. In each case we study the effect of coding.

In the block code the coded system uses (15,11) BCH code that can correct up to one error in each 15 bit codeword. A convolutional code of constraint length 7, minimum distance 10 and rate 1/2 is considered, and hard decision Viterbi decoding is assumed. We consider the performance of a rate 1/2 , turbo code for two different interleavers of size k=1000, w=3, and dTCw,min =9.

The probability of error for each satellite for the three users' distributions are shown in figures (2), (3) and (4). It is clear from these figures that the turbo code outperforms all other coding techniques for the three satellites. A probability of error of 10-7 or less can be obtained depending on the uniformity of the traffic.

From the above three figures, in the uniform case (ω>5) a slight improvement from 0.027 to 0.018 in case of block code has been obtained. However a value of 3x10-5 and 2x10-11 has been obtained for convolutional code and turbo code respectively.

The effect of coding on the capacity for each satellite in the case of one, two and three populated area is investigated in figures (5), (6) and (7). Using the turbo code, the capacity of the satellite increases 4 to 5 times or more depending on the uniformity of the traffic. Furthermore block code gives slight improvement where as convolutional, and turbo code give larger improvements.

7. CONCLUSIONS:

A Gaussian mixture traffic model was introduced, it was shown that this distribution can fit specific cases of the globe, by the proper choice of the parameters εi and μ. Previous model is considered as a special case of this suggested model. In the uniform case we conclude that the probability of error decreases from 0.027 to 0.018, 3x10-5 and 2x10-11 using block, convolutional and turbo coding respectively. Furthermore, the capacity increases from 14 to 21, 83 and 100. Slight varitions around these values are obtained depending on the traffic nonuniformity. Thus Turbo coding improves the performance of the LEO satellite system to a great extend.

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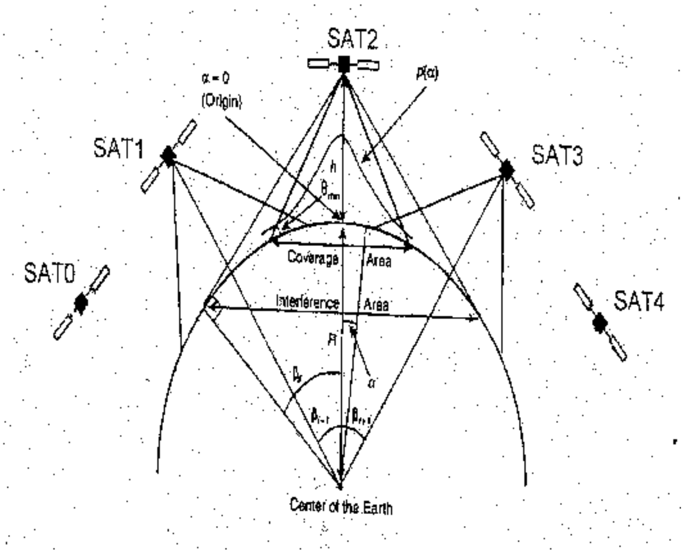

Figure (1): *The system model*

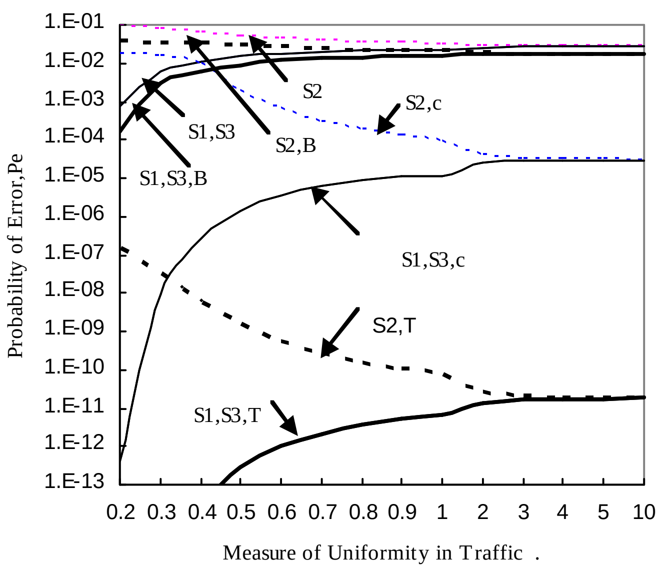

Figure (2): *Probability of error in the case of one populated area without coding, with block code (B), with convolutional code (C) and with Turbo code (T) for the three adjacent satellites (S1, S2, S3)*

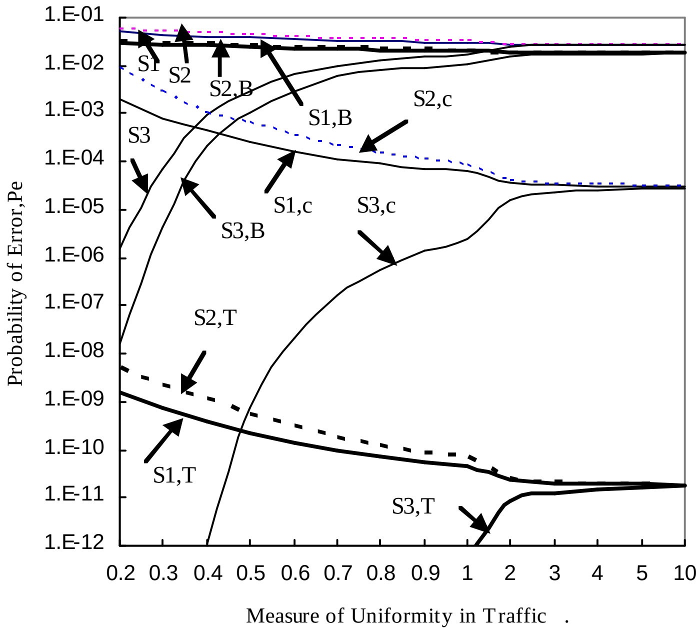

Figure (3): *Effect of coding on the probability of error in the case of two populated areas*

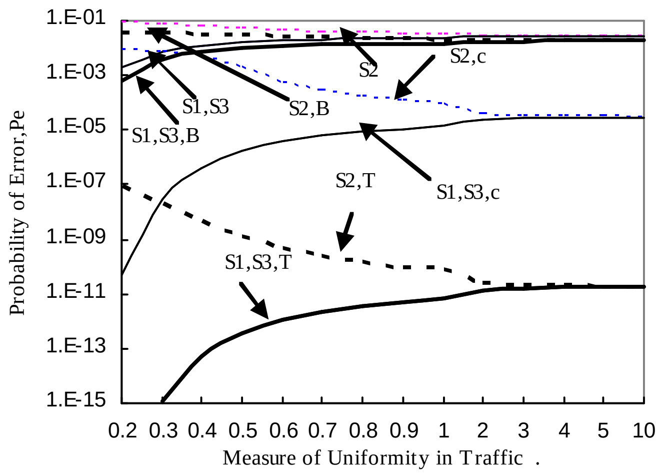

Figure (4): *Effect of coding on the probability of error in the case of three populated areas*

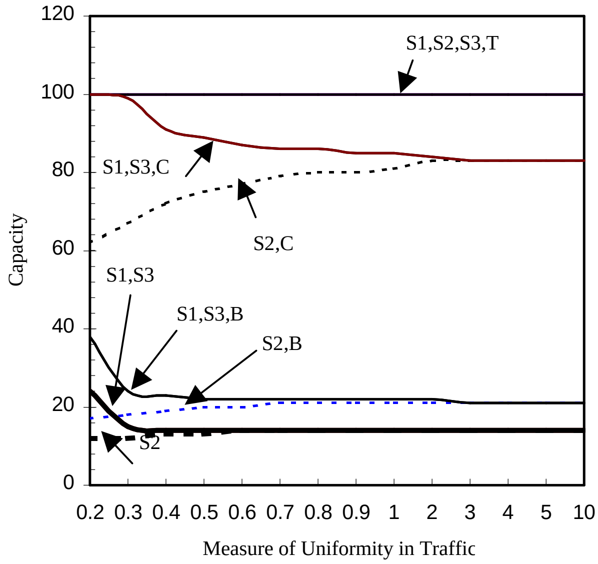

Figure (5): *Effect of coding on the capacity in the case of one populated area*

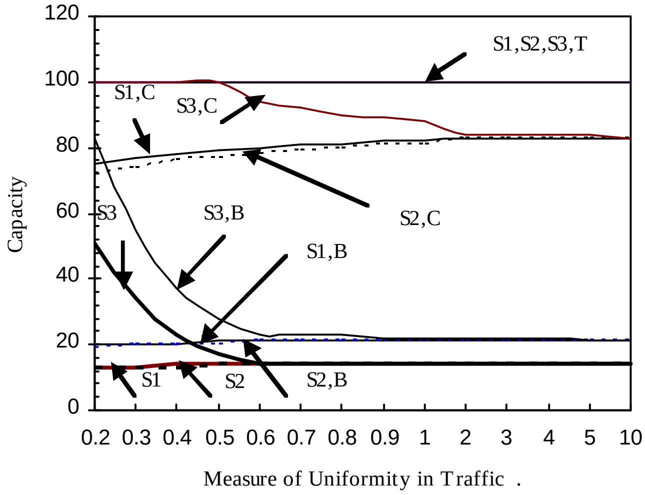

Figure (6): *Effect of coding on the capacity in the case of two populated areas*

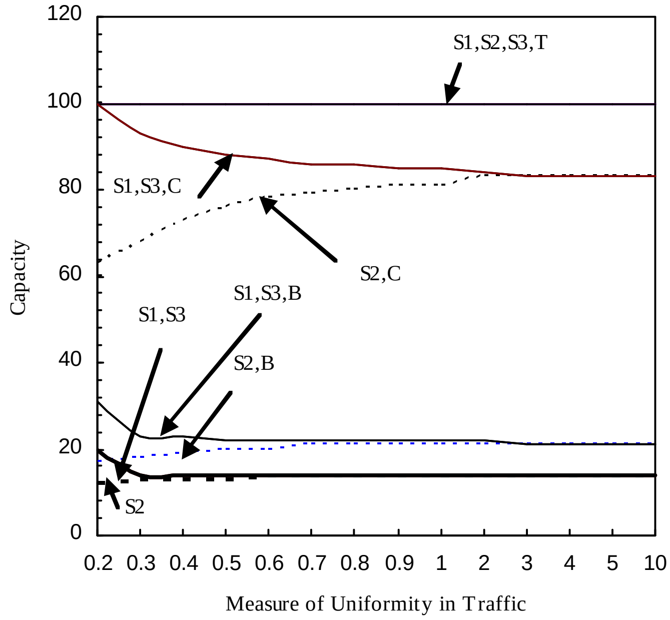

Figure (7): *Effect of coding on the capacity in the case of t*