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PARETO OPTIMAL SYNTHESIS OF THE LINEAR ARRAY GEOMETRY FOR MINIMUM SIDELOBE LEVEL AND NULL CONTROL DURING BEAM SCANNING

By

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Abstract:

In this work, synthesis of the linear array geometry is put forward as a constrained vector optimization problem whose components are to meet the minimum sidelobe level (SLL) and control of the wide/narrow null placement during beam scanning. Since these synthesis objectives generally conflict with each other, non-dominated solutions are searched using the Non-Dominated Sorting Genetic Algorithm- II (NSGA-II). Then, the Pareto frontiers are obtained using these trade-off solution sets between the maximum SLL, null control and scanning range to provide a view of all design options. Thus, the pattern features resulted from these Pareto frontiers are valid for any chosen main beam direction within its full prescribed beam scanning range. Finally, some typical Pareto optimal radiation patterns of the scanning arrays are synthesized with only perturbating the positions of the array elements and their full electromagnetic wave simulations are also completed to examine the resulted mutual coupling effects between the elements of the arrays.

Keywords:

Linear array, sidelobe suppression, null control, beam scanning, constrained multi-objective optimization, Pareto-optimal solutions.

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1. Introduction:

Mathematically, the synthesis of an antenna array is a highly nonlinear multi-objective optimization problem whose objectives are generally conflicting with each other. As far as the linear antenna array is concern, among the main objectives, gain of the main beamwidth, minimum SLL and narrow/wide null control during beam scanning can be considered. In literature heretofore, many synthesis methods have been experienced by considering each objective individually for a chosen fixed main beam direction: In the works of [1-2], the methods are concerned with suppressing the SLL while preserving the gain of the main beam. Other methods deal with only the interference suppression and null control [3]. However, typically in [4], sidelobe suppression and null control are taken together into account with the maximum radiation in broadside, employing a novel stochastic algorithm, "particle swarm optimization". On the other hand, in the computational science, different strategies are used for solving this type of classical multi-objective optimization problems. On one hand, the decision-making is reduced into a single objective, using weights for each objective, and scalar optimization is used to find the corresponding solution. Such approach entails repetition of the procedure of assigning weights until a satisfactory solution is found. Thus, the decision-making is applied at the end of the optimization procedure. Another approach is to find the Pareto optimal frontier which contains all the Pareto optimal solutions [5]. A Pareto optimal solution is a non-dominated solution which, by definition, the best that can be achieved for one objective without disadvantaging at least one other objective.

The basic advantages of an evolutionary algorithm in multi-objective optimization are: (1) It can generate a population of efficient solutions (non-dominated solutions) in a single run and (2) It eliminates the use of weighted parameters or aggregation functions. It has also become a preferred method for multi-objective optimization problems that are too complex for traditional techniques.

Among the typical works on the electromagnetic synthesis that uses the MOEAs, the works [6] can be considered which employ the particle swarm, genetic and ant colony evolutionary algorithms.

The goal of this work is to synthesize the linear antenna array geometry that is to determine the physical layout of the array employing the Pareto optimal frontiers, based on the required objectives for the radiation pattern. As is well-known, the beam scanning within the required range is one of the most significant properties expected from an array. During beam scanning, providing the MSLL at its possible minimum level and controlling nulls are inevitably demanded. In this work, these objectives are achieved as non-dominated solutions only by the perturbations of the array elements' positions in a linear geometry, while keeping a uniform excitation over the array aperture and with good quality directivity. Here the NSGA-II [5] is used in forming the Pareto optimal solutions as a fast non-dominated genetic sorting algorithm. Then, the

Pareto frontiers are obtained using the trade-off solution sets between the MSLLs, null control and scanning range to provide a view of all design options. In order to minimize mutual coupling effects between the elements, the minimum inter-element spacing is obeyed in the array geometry synthesis process and the full electromagnetic wave simulations of the worked example are also presented which verify the synthesized radiation patterns.

2. Multi-Objective Optimization and Pareto Optimal Solution Sets:

Multi-objective optimization is the process of the simultaneous minimization or maximization of m objective functions $\vec{f}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), \ldots\ldots\ldots, f_m(\vec{x}))$ with respect to an n decision variables $\vec{x} = (x_1, x_2, x_3, \ldots, x_n)$ subject to the given constraints in the decision space *X*. Thus, the multiple-objective function $\vec{f}: X \Rightarrow Y$ evaluates the quality of the specific solution by assigning an objective vector $\vec{y} = (y_1, y_2, y_3, \ldots, y_m)$ in the *Y* –objective space:

$$\text{Minimize/maximize,} \quad f_i(\vec{x}) \qquad i = 1, 2, \ldots, m. \qquad (1.a)$$

$$\text{subject to} \quad g_i(\vec{x}) \geq 0\,; \qquad i = 1, 2, \ldots, j, \qquad (1.b)$$

$$h_i(\vec{x}) = 0 \qquad i = 1, 2, \ldots, k, \qquad (1.c)$$

$$x_{i_l} \leq x_i \leq x_{i_u} \qquad i = 1, 2, \ldots, n. \qquad (1.d)$$

The constraints given by (1.b), (1.c) and (1.d) determines a feasible *D* region in the decision space $x \in R^n$, and the multiple-objective function $\vec{f}$ maps this feasible region *D* into an objective function space $y \in R^n$. If any of the objective functions $f_i(\vec{x})$, *i*=1,2,….,m are conflicting, no single objective vector dominates another objective vector. Instead, the outcome is a set of solutions and the concept of non-domination or Pareto optimality must be used to characterize the objectives [5]. A non-dominated solution is an optimal solution if it is not dominated by another solution. The set of Pareto optimal solutions reflects the trade-off surfaces between the different objectives. This set of Pareto optimal solutions is referred to as the Pareto frontiers.

In this work, the evolutionary multi-objective optimization procedure we used is denominated NSGA-II proposed by Deb et al. [5]. In the work of Deb and et al. [5], a fast non-dominated sorting genetic algorithm is presented that alleviates all the main difficulties of the previous non-dominated sorting multi-objective evolutionary algorithms. Specifically, *O(MN3)* computational complexity where *M* is the number of objectives and *N* is the population size, is overcome by a fast non-dominated sorting approach. Simulation results on difficult test problems show that the NSGA-II, in most problems, is able to find much better spread of solutions and better convergence near the true Pareto optimal frontier compared to Pareto-archived evolution strategy and strength Pareto evolutionary algorithms. Thereby, we chose this algorithm for its ease of implementation and its efficient computation of non-dominated ranks within high degree of accuracy. The detailed theory together with literature can be found in [5].

2. Problem Formulation:

In this work, a linear phased array of 2*N* isotropic elements placed symmetrically along *y*-axis is considered as shown in Fig. 1. Due to the symmetry of the array geometry, dependence of the array factor in the azimuth plane can be expressed as:

$$AF = 2\sum_{n=1}^{N} A_n \cos\left[k\, d_n \sin\phi + \beta_n\right] \tag{2}$$

where k is the wavenumber and A_n, B_n and d_n are the excitation amplitude, phase and location of the nth element, respectively. If it is assumed that the maximum radiation of the array is required to be oriented at an angle $\phi_o\,(-90^o \le \phi_o \le 90^o)$, the phase excitation β_n of the n th element must be equal to [7]:

$$k\, d_n \sin\phi + \beta_n\big|_{\phi=\phi_o} = 0 \Rightarrow \qquad \beta_n = -k\, d_n \sin\phi_o \tag{3}$$

Thus, by controlling the progressive phase difference between the elements, the maximum radiation can be squinted in any required direction to form a scanning array. If β_n in (3) is placed in (2) the array factor can be obtained as follows:

$$AF = 2\sum_{n=1}^{N} A_n \cos\left[k\, d_n\, (\sin\phi - \sin\phi_0)\right] \tag{4}$$

In this work, since the maximum levels of the sidelobe regions are to be minimized and narrow/wide nulls in any required locations are generated by means of perturbations in the elements' positions, by introducing Δ_n displacement to the n th element, Eq. (4) becomes:

$$AF = 2\sum_{n=1}^{N} A_n \cos\left[k\,(d_n + \Delta_n)\,(\sin\phi - \sin\phi_0)\right] \tag{5}$$

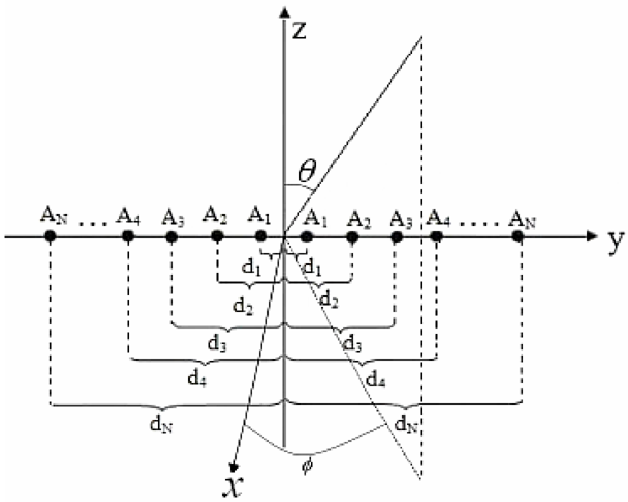


Figure (1): *Linear antenna array geometry*

Now the statement of the problem simply reduces to (5) the use of Pareto Genetic Algorithm (PGA) to find the perturbation amount Δ_n s of the array elements that will result in an array beam with a narrow/wide null generations in the required directions and minimization of the maximum levels of sidelobe regions of the radiation pattern while the main beam is scanning within the prescribed ranges.

<u>4. Pareto Genetic Optimal Synthesis of the Linear Phased Antenna Arrays:</u>

In this work, we are interested in synthesis of the geometry of a linear antenna array with the minimum SLL and generating narrow/wide nulls at the interference locations while scanning main beam between the required scan angles. Thus, the problem statement can be arranged as follows:

$$Minimize\left\{f_1(\vec{\Delta})\right\} = Minimize\left\{\max\left[\max\left|AF(\phi,\vec{\Delta})\right|_{\phi=-90^o}^{\phi=\phi_o-\frac{\phi_{BW}}{2}},\quad \max\left|AF(\phi)\right|_{\phi=\phi_o+\frac{\phi_{BW}}{2}}^{\phi=90^o}\right]_{\phi_o=\phi_{ol}}^{\phi_o=\phi_{ou}}\right\} \tag{6}$$

$$Minimize\left\{f_2(\vec{\Delta})\right\} = Minimize\left\{\max\left[\max\left|AF(\phi,\vec{\Delta})\right|_{\phi=\phi_{null_l}}^{\phi=\phi_{null_u}}\right]_{\phi_o=\phi_{ol}}^{\phi_o=\phi_{ou}}\right\} \tag{7}$$

$$\text{subject to} \quad 0.1\lambda \le \left|\Delta_n\right| \tag{8}$$

where ϕ_o and ϕ_{BW} are the direction of the maximum radiation and the corresponding main beamwidth, respectively; while ϕ_{o_l} and ϕ_{o_u} stand for the lower and upper boundaries of the scanning angles. In both (6), (7) the minimization operator is applied to the maximum values chosen between all the greatest values within the sidelobe and null regions during main beam scanning. (8) gives the limitations of the perturbation amount to minimize the mutual coupling effects between the elements.

5. Typical Examples for the Pareto Optimal Synthesis of Linear Phased Arrays:

In this section, some typical examples resulted from the application of Pareto optimization using the enhanced NSGA II to the objectives stated by the (6), (7) and (8) are given. In the worked examples, a uniformly excited linear antenna array is considered that has 40 isotropic elements with initially uniform inter-element spacing of 0.5 λ. Hereafter, this original array configuration is named as "Conventional" array. The second antenna array to be considered is the thinned version of the conventional array, which is obtained by a simple genetic optimization by rounding the excitation amplitudes either 1 or 0 to minimize the maximum of SLL during the beam scanning within the prescribe region as stated the objective function in (6).

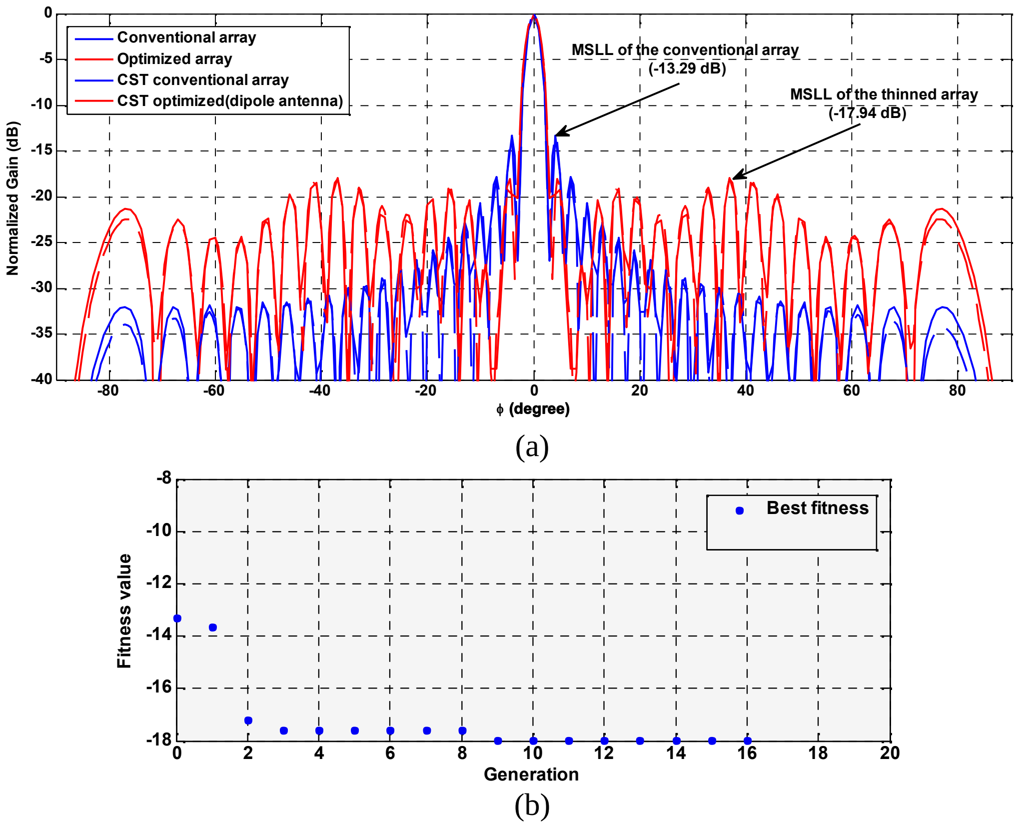
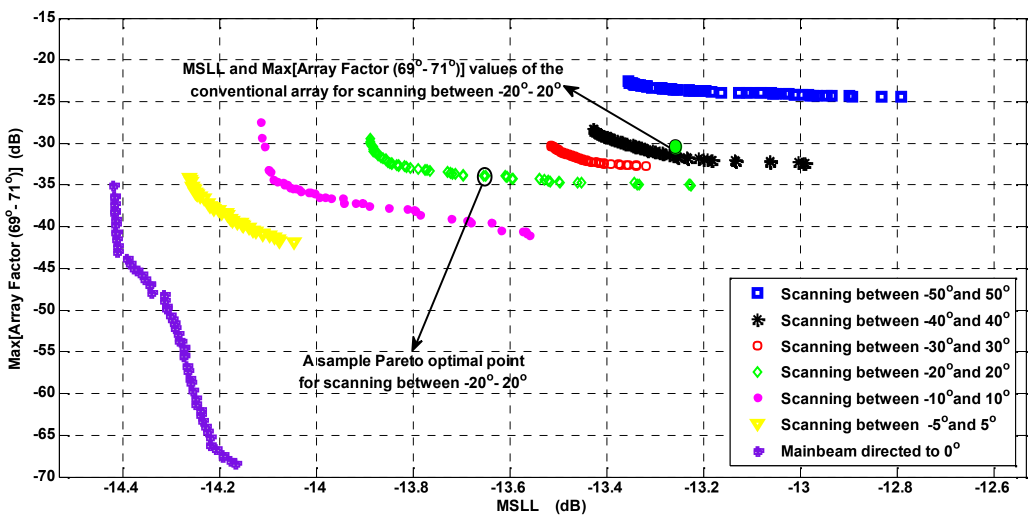
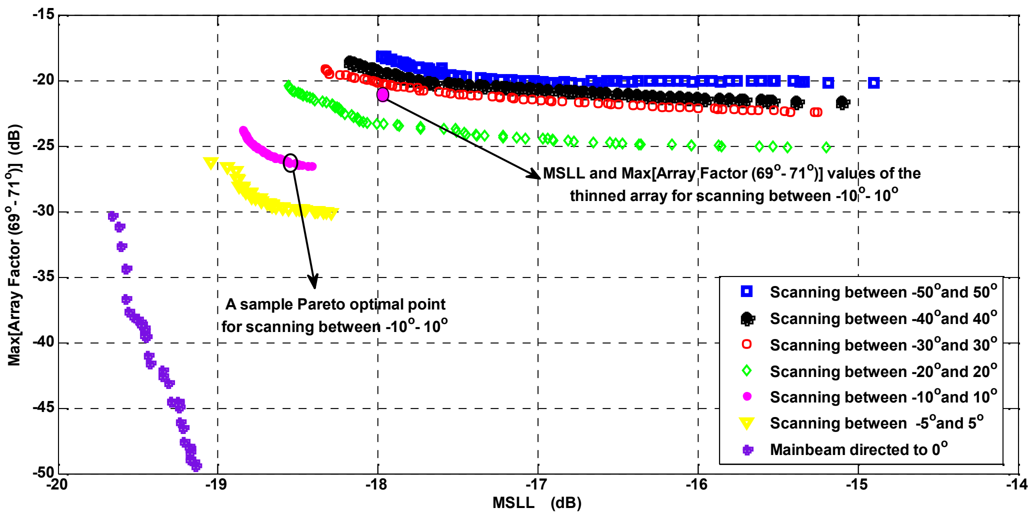


Figure (2): *Normalized pattern of the 34-element thinned conventional array in the symmetrical configuration as compared with the patterns of the 40-element conventional array and the full-wave simulation*

The resulted radiation pattern is given compared as the conventional and the "full – electromagnetic wave" –CST- simulation patterns in Fig. 2(a), which is obtained only after 9 generations as seen from its convergence characteristic in Fig. 2(b). Hereafter, all the radiation patterns synthesized by "Pattern Multiplication" are verified by their corresponding full-wave simulations using half–wave dipoles at 2.6 GHz. In Fig. 2(a), it can be seen that approximately 5 dB reduction in the conventional MSLL is achieved by turning off the appropriate six elements of the conventional array that results in a decrease of necessity for energy, cost and complexity of the array.



(a)



(b)

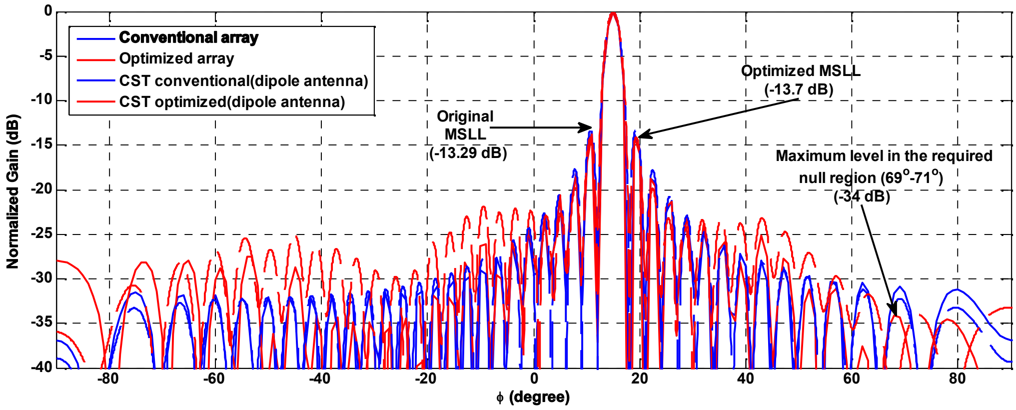
Figure (3): *Pareto frontiers of the null region within the interval 69° -71° for the 40-element conventional array for the given scanning ranges: (a) The original; (b) The thinned arrays. Non–optimized solutions are taken place in the single isolated points*

Figs. 2(a) and 2(b) show the Pareto frontiers of the conventional and thinned arrays, respectively, for the scanning ranges of [(-50°)- 50°], [(-40°)- 40°], [(-30°)- 30°], [(-20°)- 20°], [(-10°)- 10°], [(-5°)- 5°] and [0°] for the case of a narrow null between 69^0 - 71^0.

Besides, Table I gives all the corresponding non-optimized solution pairs, some with the stars (*) of which are shown as the highlighted isolated single points in the Pareto frontiers' planes. Thus, the Pareto frontiers of the trade-off solution sets resulted from NSGA-II in Fig. 3(a) and 3(b) provide a view of all design options for the predetermined cases. In the next stage a decision must be made for choosing the solution set to be synthesized depending on the application.

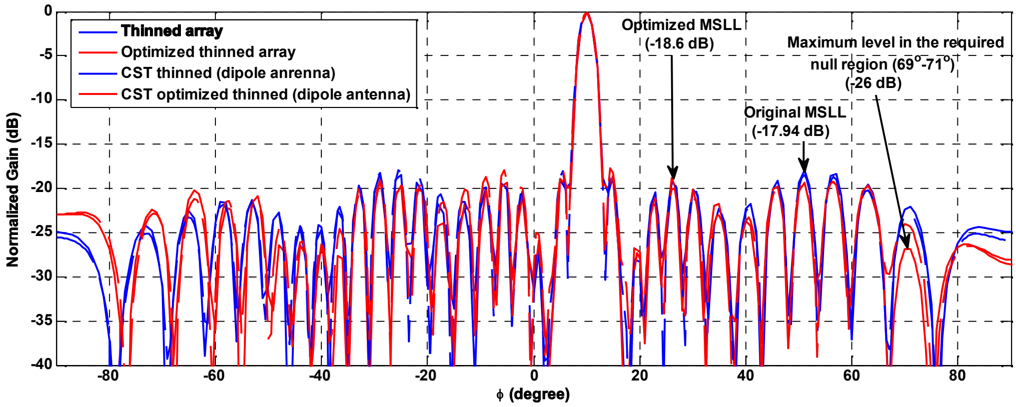
Table (1): *Positions of the non-optimized points in the plane of the maximum levels of the whole side lobe and the null region within the interval 69° -71°*

	Conventional Array		Thinned Array	
	MSLL	max[AF(69^0-71^0)]	MSLL	max[AF(69^0-71^0)]
Main beam directed to 0°	-13.30 dB	-33.32 dB	-17.96 dB	-25.7 dB
Scanning between [(-5^0) - 5^0]	-13.29 dB	-31.88 dB	-17.94 dB	-22.55 dB
Scanning between [(-10^0) - 10^0]	-13.29 dB	-31.57 dB	-17.94* dB	-22.11* dB
Scanning between [(-20^0) - 20^0]	-13.29* dB	-30.43* dB	-17.94 dB	-17.94 dB
Scanning between [(-30^0) - 30^0]	-13.29 dB	-28.71 dB	-17.94 dB	-17.94 dB
Scanning between [(-40^0) - 40^0]	-13.24 dB	-25.81 dB	-17.94 dB	-17.94 dB
Scannng between [(-50^0) - 50^0]	-13.24 dB	-20.68 dB	-17.94 dB	-17.94 dB



(a)

Figure (4): *Normalized radiation patterns for the conventional array, optimized array and CST simulation of the half-wave dipoles for the chosen Pareto optimal point in Fig.3a. Here the main beam direction is chosen as 15°*



(a)

Figure (5): *Normalized radiation patterns for the thinned array, optimized array and CST simulation of the half-wave dipoles for the chosen Pareto optimal point in Fig.3b. Here the main beam direction is chosen as 10°*

As it can be observed that from the Figs. 3(a) and 3(b), increase in the scanning range follows decrease in the reduction performance of the MSLL and maximum level within the null region of the linear array. The radiation pattern in Fig. 4, illustrates the synthesis of a 40-element linear array directed to 15° for the chosen point (-13.7dB; -34 dB) within the scanning range [(-20°) – 20°] which corresponds to the Pareto optimized version of the highlighted original point in the Pareto frontier plane in Fig.3a and the radiation pattern in Fig. 5, illustrates the synthesis of a 40-element linear array directed to 10° for the chosen point in Figure 3(b). Besides the radiation patterns of the conventional and CST simulation version of the Pareto optimized patterns are also given. Half- power beam widths are nearly constant and $\phi_{HPBW} = 2.625^o$ for all the patterns and the Table (2) gives the necessary displacements of the elements for the synthesized radiation pattern.

Table (2): *Optimized element position perturbations Δ_n for Fig. 3(a), 3(b)*

Element Number	Δ_n (in λ)		Element Number	Δ_n (in λ)	
	Figure 3(a)	Figure 3(b)		Figure 3(a)	Figure 3(b)
± 1	0.0045	0.0237	± 11	-0.0352	0.0356
± 2	-0.0200	0.0455	± 12	-0.0446	0.0512
± 3	-0.0118	0.0397	± 13	-0.0232	0.0596
± 4	-0.0065	0.0135	± 14	0.0030	0.0541
± 5	-0.0020	0.0540	± 15	-0.0035	-
± 6	-0.0430	0.0277	± 16	-0.0030	-
± 7	-0.0489	0.0329	± 17	0.0289	0.0271
± 8	-0.0399	0.0412	± 18	0.0579	0.0952
± 9	-0.0217	0.0185	± 19	0.0080	-
± 10	-0.0290	0.0312	± 20	0.0410	0.0355

6. Conclusions:

In this work, synthesis of the scanning linear antenna array is put forward as a constrained vector optimization problem whose components are to meet the minimum SLL, wide/ narrow null placement during beam scanning . The NSGA-II is used in forming the Pareto optimal solutions. The thinned antenna array version is also obtained using a genetic optimization to minimize the MSLL during beam scanning within the prescribe region as stated the multi-objective function given by (6). Finally the proposed method is applied to the synthesis of some linear antenna arrays using only perturbations in the elements' positions, based on the requirements on the chosen Pareto optimal points. Furthermore the full electromagnetic wave simulations of these antenna arrays are also completed and thus, the mutual coupling effects are examined. It can be concluded that the proposed method is successful to meet the Pareto optimal requirements even if with only perturbations in the elements' positions.

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