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Tuning of Digital PID Controller for Multivariable Systems Using Local Optimal Controller

By

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Abstract:

This paper introduces a new tuning technique for digital PID controller parameters of multivariable systems. This technique is based on the modified Local Optimal Controller (LOC) parameters for certain predefined model structures. The modified LOC parameters can be obtained using the identified parameters of the predefined model structure. As such, the digital PID controller parameters can be tuned using the model parameters of certain predefined structures and a single tunable parameter related to the LOC for each output of the multivariable system. Becoming a model-based controller, the PID controller parameters can be adjusted in automatic mode.
This new technique is compared with the existing Genetic Algorithm (GA) technique for tuning digital PID controller parameters. The comparison is based on the experimental results of the Bytronic Process Control Unit (PCU).

Keywords:

Digital PID Tuning, Local Optimal Controller, Multivariable Systems.

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1. Introduction:

Due to the increasing complexity of process control systems, multivariable process control has been received considerable attention over the past decades and numerous theoretical and practical studies have been done in this area of research [1]. Among the existing methods of multivariable control, the LOC is a method introduced by Lyantsev, et al. in 2004 [2]. However, this LOC approach is incapable of dealing with non-minimum phase systems.
A modified LOC approach for multivariable systems is proposed By Ashry, et al. in 2008 [3].
The proposed method provides closed loop stability when dealing with non-minimum phase plant, which is a considerable advantage over the original LOC.
As discussed in [2, 3], LOC is a model-based controller. The model parameters and a tunable parameter (h) for each output of the multivariable system are used to design that controller. The LOC is designed for reduced order models as well as for full order models and an excellent controlled system performance is achieved [4, 5].
Based on the results obtained for reduced order LOC, certain predefined model structures are used to introduce a new tuning technique of digital PID/PI for Single-Input Single-Output (SISO) systems [6, 7].
In this paper, a new tuning technique for digital PID controller parameters of multivariable systems is introduced. This technique is based on the modified LOC parameters for certain predefined model structures.
General relations between the multivariable PID controller parameters and the modified LOC parameters are deduced in section 2. In section 3, the detailed relations are deduced for Two-Input Two-Output (TITO) system. The experimental results are represented in section 4. The new tuning technique is applied to Bytronic PCU as a TITO system. The results obtained are compared with those of a conventional tuning method (GA-tuned PID). Finally, the conclusion remarks are represented in section 5.

2. Tuning of digital PID controller using LOC for MIMO systems:

In this section, relations between the multivariable PID controller parameters and the modified LOC parameters, which are functions of model parameters, are deduced for certain predefined model structures. As such, the multivariable PID controller parameters are transformed into model parameters and one tunable parameter for each output.

2.1 Digital PID controller for multivariable systems:

In this subsection, a multivariable system of *n*-inputs and *m*-outputs will be considered

(i.e. $n \times m$ system). The block diagram of this multivariable system when controlled by PID controller is shown in Figure (1), where:
R is the reference input vector for the closed loop system ($m \times 1$),
U is the PID controller output vector or the plant input vector ($n \times 1$),
Y is the closed loop system output vector ($m \times 1$),
E is the tracking error vector ($m \times 1$), $E = R - Y$.
These vectors are defined in (1).

Figure (1): *The block diagram of multivariable system controlled by PID.*

$$R = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix}, \quad U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}, \quad E = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} \tag{1}$$

The multivariable PID controller (C) is $n \times m$ matrix of the form given in (2), where PID_{ij} is given in (3).

$$C = \begin{bmatrix} PID_{11} & PID_{12} & \cdots & PID_{1m} \\ PID_{21} & PID_{22} & \cdots & PID_{2m} \\ \vdots & \vdots & & \vdots \\ PID_{n1} & PID_{n2} & \cdots & PID_{nm} \end{bmatrix}, \tag{2}$$

$$PID_{ij} = \frac{(K_{Pij} + K_{Iij} + K_{Dij}) - (K_{Pij} + 2K_{Dij})z^{-1} + K_{Dij}z^{-2}}{1 - z^{-1}} \tag{3}$$

The controller output vector U can be represented as in (4). As such, $u_k(i)$ *is given* in (5).

$$\begin{bmatrix} u_1(i) \\ u_2(i) \\ \vdots \\ u_n(i) \end{bmatrix} = \begin{bmatrix} PID_{11} & PID_{12} & \cdots & PID_{1m} \\ PID_{21} & PID_{22} & \cdots & PID_{2m} \\ \vdots & \vdots & & \vdots \\ PID_{n1} & PID_{n2} & \cdots & PID_{nm} \end{bmatrix} \begin{bmatrix} e_1(i) \\ e_2(i) \\ \vdots \\ e_m(i) \end{bmatrix} \tag{4}$$

$$u_k(i) = PID_{k1}e_1(i) + PID_{k2}e_2(i) + \cdots + PID_{km}e_m(i) \tag{5}$$

Using equation (3), equation (5) can be represented as in (6).

$$\begin{aligned} \delta u_k(i) = {} & (K_{Pk1} + K_{Ik1} + K_{Dk1})e_1(i) - (K_{Pk1} + 2K_{Dk1})e_1(i-1) + K_{Dk1}e_1(i-2) \\ & + (K_{Pk2} + K_{Ik2} + K_{Dk2})e_2(i) - (K_{Pk2} + 2K_{Dk2})e_2(i-1) + K_{Dk2}e_2(i-2) \\ & + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ & + (K_{Pkm} + K_{Ikm} + K_{Dkm})e_m(i) - (K_{Pkm} + 2K_{Dkm})e_m(i-1) + K_{Dkm}e_m(i-2) \end{aligned} \tag{6}$$

where $\delta u_k(i) = u_k(i) - u_k(i-1)$.

2.2 LOC for multivariable systems of predefined model structure:

To be compared with PID controller, the multivariable model structure used to design the LOC is represented in (7), where G_{jk} is given in (8).

$$\begin{bmatrix} y_1(i) \\ y_2(i) \\ \vdots \\ y_m(i) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \begin{bmatrix} u_1(i) \\ u_2(i) \\ \vdots \\ u_n(i) \end{bmatrix} \tag{7}$$

$$G_{jk} = \frac{b_{jk}z^{-1}}{1 + a_{j1}z^{-1} + a_{j2}z^{-2}} \tag{8}$$

As such, the following equation can be represented for each output $y_j(i)$.

$$y_j(i) = -a_{j1}y_j(i-1) - a_{j2}y_j(i-2) + b_{j1}u_1(i-1) + b_{j2}u_2(i-1) + \cdots + b_{jn}u_n(i-1) \tag{9}$$

From this model output equation the following equation can be deduced for the LOC [2, 3].

$$b_{j1}\delta u_1(i)+b_{j2}\delta u_2(i)+\cdots+b_{jn}\delta u_n(i) = \frac{1}{h_j}e_j(i)+a_{j1}y_j(i)+(a_{j2}-a_{j1})y_j(i-1)-a_{j2}y_j(i-2) \quad (10)$$

For constant reference input vector *R*, the following equality is used to produce (12).

$r_j(i) = r_j(i-1) = r_j(i-2)$ (11)

$$b_{j1}\delta u_1(i)+b_{j2}\delta u_2(i)+\cdots+b_{jn}\delta u_n(i) = (\frac{1}{h_j}-a_{j1})e_j(i)+(a_{j1}-a_{j2})e_j(i-1)+a_{j2}e_j(i-2) \quad (12)$$

For $n\times n$ systems, the *n* equations produced from (12) are solved to produce the following group of equations.

$$\begin{bmatrix}\delta u_1(i)\\ \delta u_2(i)\\ \vdots\\ \delta u_n(i)\end{bmatrix} = \begin{bmatrix} f_1(e_1(i),e_1(i-1),e_1(i-2),\cdots,e_n(i),e_n(i-1),e_n(i-2))\\ f_2(e_1(i),e_1(i-1),e_1(i-2),\cdots,e_n(i),e_n(i-1),e_n(i-2))\\ \vdots\\ f_n(e_1(i),e_1(i-1),e_1(i-2),\cdots,e_n(i),e_n(i-1),e_n(i-2))\end{bmatrix} \quad (13)$$

2.3Multivariable PID controller parameters in terms of LOC parameters:

To represent the multivariable PID controller parameters in terms of the LOC parameters, the coefficients of $e_1(i)$, $e_1(i-1)$, $e_1(i-2)$, . . . , $e_n(i)$, $e_n(i-1)$, $e_n(i-2)$ are equated in (13) and (6).

As such the multivariable PID controller parameters K_{Pij} ,K_{Iij} ,K_{Dij} are functions of the multivariable model parameters and the *n* tunable parameters $h_{1,2,\ldots,n}$.

In the next section, the multivariable PID controller parameters will be deduced in details for the case studied (2×2 system).

3. Multivariable PID controller parameters in terms of LOC parameters for two-input two-output systems:

For 2×2 systems, the multivariable PID controller outputs can be represented in the following two equations.

$$\begin{aligned}\delta u_1(i) &= (K_{P11}+K_{I11}+K_{D11})e_1(i)-(K_{P11}+2K_{D11})e_1(i-1)+K_{D11}e_1(i-2)\\ &+(K_{P12}+K_{I12}+K_{D12})e_2(i)-(K_{P12}+2K_{D12})e_2(i-1)+K_{D12}e_2(i-2)\end{aligned} \quad (14)$$

$$\delta u_2(i) = (K_{P21}+K_{I21}+K_{D21})e_1(i) - (K_{P21}+2K_{D21})e_1(i-1) + K_{D21}e_1(i-2) \\ +(K_{P22}+K_{I22}+K_{D22})e_2(i) - (K_{P22}+2K_{D22})e_2(i-1) + K_{D22}e_2(i-2) \tag{15}$$

For the LOC of 2×2 systems, the following two equations are obtained.

$$b_{11}\delta u_1(i) + b_{12}\delta u_2(i) = (\frac{1}{h_1} - a_{11})e_1(i) + (a_{11}-a_{12})e_1(i-1) + a_{12}e_1(i-2) \tag{16}$$

$$b_{21}\delta u_1(i) + b_{22}\delta u_2(i) = (\frac{1}{h_2} - a_{21})e_2(i) + (a_{21}-a_{22})e_2(i-1) + a_{22}e_2(i-2) \tag{17}$$

Solving these two equations to produce the group of equations represented in (13), the following two equations are deduced.

$$\delta u_1(i) = \frac{1}{b_{11}(1-\frac{b_{12}b_{21}}{b_{11}b_{22}})}[(\frac{1}{h_1} - a_{11})e_1(i) + (a_{11}-a_{12})e_1(i-1) + a_{12}e_1(i-2) \\ -\frac{b_{12}}{b_{22}}(\frac{1}{h_2} - a_{21})e_2(i) + \frac{b_{12}}{b_{22}}(a_{22}-a_{21})e_2(i-1) - \frac{b_{12}a_{22}}{b_{22}}e_2(i-2)] \tag{18}$$

$$\delta u_2(i) = \frac{1}{b_{22}(1-\frac{b_{12}b_{21}}{b_{11}b_{22}})}[-\frac{b_{21}}{b_{11}}(\frac{1}{h_1} - a_{11})e_1(i) + \frac{b_{21}}{b_{11}}(a_{12}-a_{11})e_1(i-1) - \frac{b_{21}a_{12}}{b_{11}}e_1(i-2) \\ +(\frac{1}{h_2} - a_{21})e_2(i) + (a_{21}-a_{22})e_2(i-1) + a_{22}e_2(i-2)] \tag{19}$$

Considering the discrete time integrator for the modified LOC, the following two equations are deduced, where T_s is the sampling time [3].

$$\delta u_1(i) = \frac{T_s}{b_{11}(1-\frac{b_{12}b_{21}}{b_{11}b_{22}})}[(\frac{1}{h_1} - a_{11})e_1(i) + (a_{11}-a_{12})e_1(i-1) + a_{12}e_1(i-2) \\ -\frac{b_{12}}{b_{22}}(\frac{1}{h_2} - a_{21})e_2(i) + \frac{b_{12}}{b_{22}}(a_{22}-a_{21})e_2(i-1) - \frac{b_{12}a_{22}}{b_{22}}e_2(i-2)] \tag{20}$$

$$\delta u_2(i) = \frac{T_s}{b_{22}(1-\frac{b_{12}b_{21}}{b_{11}b_{22}})}[-\frac{b_{21}}{b_{11}}(\frac{1}{h_1}-a_{11})e_1(i)+\frac{b_{21}}{b_{11}}(a_{12}-a_{11})e_1(i-1)-\frac{b_{21}a_{12}}{b_{11}}e_1(i-2)$$
$$+(\frac{1}{h_2}-a_{21})e_2(i)+(a_{21}-a_{22})e_2(i-1)+a_{22}e_2(i-2)] \quad (21)$$

Finally equating the coefficients of $e_1(i)$, $e_1(i-1)$, $e_1(i-2)$, $e_2(i)$, $e_2(i-1)$, $e_2(i-2)$ in equations (14) and (15) of the PID controller from one hand and equations (20) and (21) of the LOC from the other hand, the following equations are ded*uced. These equations represent* the multivariable PID controller parameters in terms of the LOC parameters.

$$K_{P11} = -\frac{T_s}{D_1}(a_{11}+a_{12})$$
$$K_{I11} = \frac{T_s}{D_1 h_1} \quad (22)$$
$$K_{D11} = \frac{T_s}{D_1}a_{12}$$

$$K_{P12} = \frac{T_s}{D_1}[\frac{b_{12}a_{21}+b_{12}a_{22}}{b_{22}}]$$
$$K_{I12} = -\frac{T_s}{D_1}[\frac{b_{12}}{b_{22}h_2}] \quad (23)$$
$$K_{D12} = -\frac{T_s}{D_1}[\frac{b_{12}a_{22}}{b_{22}}]$$

$$K_{P21} = \frac{T_s}{D_2}[\frac{b_{21}a_{11}+b_{21}a_{12}}{b_{11}}]$$
$$K_{I21} = -\frac{T_s}{D_2}[\frac{b_{21}}{b_{11}h_1}] \quad (24)$$
$$K_{D21} = -\frac{T_s}{D_2}[\frac{b_{21}a_{12}}{b_{11}}]$$

$$K_{P22} = -\frac{T_s}{D_2}(a_{21} + a_{22})$$
$$K_{I22} = \frac{T_s}{D_2 h_2} \quad (25)$$
$$K_{D22} = \frac{T_s}{D_2} a_{22}$$

where:

$$D_1 = b_{11}[1 - \frac{b_{12}b_{21}}{b_{11}b_{22}}] \quad (26)$$

$$D_2 = b_{22}[1 - \frac{b_{21}b_{12}}{b_{11}b_{22}}] \quad (27)$$

4. Experimental results:

The laboratory-based process control system is Bytronic Process Control Unit (PCU), which is based around a fluid flow process, where either or both flow and temperature of the fluid can be controlled.

This reflects a typical process control situation such as in the food and drink manufacturing and petrochemical industry [8]. In this system a fluid is pumped in a closed path from a sump through a cooling fan to a process tank where the fluid is heated and then is drained back to the sump. The overall view of the Bytronic PCU is shown in Figure (2).

The Bytronic PCU is a 2-input 2-output system. The inputs are the voltage to the pump and the power to the heater. The outputs are the fluid flow rate and the fluid temperature.

In this section, the multivariable PID controller parameters will be tuned using the relations deduced from the LOC. As such, the model parameters for the predefined structure in (7) and (8) should be identified, then adjusting the tunable parameters h_1, h_2 the PID controller parameters are obtained.

***Figure (2)**: The overall view of Bytronic PCU test rig.*

4.1 Open loop system identification:

In this subsection; the approach explained in [3] is used to identify the multivariable model parameters. In this approach, two subsystems are investigated according to the number of outputs.

The first subsystem is SISO system. The input is the voltage to the pump, and the output is the flow rate. The second input which is the input power to the heater has no effect on the flow rate. For this subsystem, the model parameters for the second order model structure given in (8) are identified as explained in [3] using chirp and multi-sine inputs for open loop system identification.

The identified model parameters are:

$$a_{11}= -1.25, \quad a_{12}= 0.4272, \quad b_{11}= 0.0374. \tag{28}$$

The second subsystem is 2-input single-output system. The inputs are the input voltage to the pump and the input power to the heater, and the output is the fluid temperature. For this subsystem, the model parameters for the predefined model structure in (7) and (8) should be identified.

The system identification toolbox of Matlab [9] is used to identify the model parameters. The inputs for open loop system identification are chirp or multi-sine for the first input and multi-step for the second input [3]. The multi-step for the second input is due to the relatively high time constant of the heater. The model parameters are given in (29).

$$a_{21} = -0.4903, \quad a_{22} = -0.5095, \quad b_{21} = 9.3430 \times 10^{-4}, \quad b_{22} = 9.56 \times 10^{-5}. \tag{29}$$

4.2 Tuning of multivariable PID controller parameters using LOC:

Using the multivariable model parameters given in (28) and (29) and the tuning parameters (h_1=5, h_2=10), the PID controller parameters can be obtained from equations (22), (23), (24), and (25).

The multivariable PID controller parameters are listed below.

$$\begin{array}{lll} K_{P11}=2.75, & K_{I11}=0.6685, & K_{D11}=1.4278, \\ K_{P12}=0, & K_{I12}=0, & K_{D12}=0, \\ K_{P21}=-26.8758, & K_{I21}=-6.5328, & K_{D21}=-13.9540, \\ K_{P22}=1307.2699, & K_{I22}=130.72, & K_{D22}=-666.1872. \end{array} \tag{30}$$

Figure (3) and Figure (4) show the two output responses of the PID controlled system when the controller parameters used are those obtained above from the model-based LOC. In these figures, the reference input for the flow rate is changed from *0.8 to 1.2 l/min* at $t = 500s$ and the reference input for the fluid temperature is changed from room temperature to 60^oC at $t = 20s$. The oscillations for the output temperature (60 ± 1) are due to the sensor sensitivity.

4.3 System control using genetically tuned PID:

To design PID controller for the multivariable system under consideration, a full matrix of digital PID controllers is used as expressed in (31).

$$C(z) = \begin{bmatrix} K_{P11} + \dfrac{K_{I11}}{1-z^{-1}} + K_{D11}(1-z^{-1}) & K_{P12} + \dfrac{K_{I12}}{1-z^{-1}} + K_{D12}(1-z^{-1}) \\ K_{P21} + \dfrac{K_{I21}}{1-z^{-1}} + K_{D21}(1-z^{-1}) & K_{P22} + \dfrac{K_{I22}}{1-z^{-1}} + K_{D22}(1-z^{-1}) \end{bmatrix} \tag{31}$$

Figure (3): *The first output response (flow rate) of the multivariable system controlled by Digital PID tuned using model-based LOC.*

Figure (4): *The second output response (fluid temperature) of the multivariable system controlled by digital PID tuned using model-based LOC.*

A GA program is used to tune the twelve controller parameters represented in (31) through minimizing the Mean Squared Error (MSE) between the reference input and the system's output and to have no overshoot as multi-objective function [10–12]. These twelve controller parameters are listed in (32).

$$
\begin{aligned}
&K_{P11} = 3.0831, \quad K_{I11} = 1.0109, \quad K_{D11} = 5.0335;\\
&K_{P12} = 4.6583, \quad K_{I12} = 0.0064, \quad K_{D12} = 2.6325;\\
&K_{P21} = 0.7092, \quad K_{I21} = -5.8304, \quad K_{D21} = -0.0541;\\
&K_{P22} = 0.6151, \quad K_{I22} = 8.5757, \quad K_{D22} = -2.6268.
\end{aligned} \qquad (32)
$$

Figure (5) and Figure (6) show the two output responses for the real system using genetically tuned PID controller, where the reference input for the flow rate is changed from *0.8* to *1.2 l/min* at *t = 500s* and the reference input for the fluid temperature is changed from room temperature to 60^oC at *t=20s*. The oscillations for the output temperature (*60±1*) are due to the sensor sensitivity.

Figure (5): *The first output response of the multivariable system controlled by the genetically tuned PID.*

Figure (6): *The second output response of the multivariable system controlled by the genetically tuned PID.*

From these figures, the output responses obtained are similar to those obtained previously in Figure (3) and Figure (4). The advantage of tuning multivariable PID controller parameters using LOC over the genetically tuned PID is that the multivariable PID controller can be used in automatic mode based on the on-line model parameters identification.

5. Conclusions:

Based on LOC, a new tuning technique of digital PID controller parameters is introduced for multi-variable systems. Certain predefined model structures are used to deduce the relations between PID controller and LOC.

For multivariable systems, the model-based LOC is used to find the multivariable PID controller parameters. The results obtained are similar to those obtained using GA-tuned PID. The experimental results confirm the effectiveness of the new tuning technique.

As the multivariable PID controller parameters in this tuning technique are functions of the model parameters, which are constants, and one tunable parameter h for each output; this tuning method reduces the tuning parameters of the PID controller. For $n \times n$ system, the tuning parameters for conventional tuning methods are $3n^2$. These tuning parameters are reduced to only n using the new tuning technique.

This tuning method transforms the PID controller to a model-based controller, so it can be used in automatic adaptation mode based on the on-line system identification.

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