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PERFORMANCE OF A LOW BIAS DF ESTIMATOR IN WIDE BAND COHERENT SOURCES ENVIRONMENT

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Abstract

The estimation of radio activities location is a main task of radio reconnaissance systems. The performance measures of the estimators of these systems are usually the resolution threshold, bias, and variance of the estimated directions. Recently, a low bias estimator, named D_r , is proposed for applications in narrow band incoherent sources environment. In this paper, some attractive features of the D_r in a practical environment of wide band coherent sources are introduced. It also presents a process, based on the D_r for attaining a low resolution threshold with a low level of both bias and variance.

Key Words

Direction Finding, DSP

1. Introduction

In Direction Of Arrivals (DOA's) estimation problems, the variance, bias, and resolution threshold are the measures of the estimator performance. Most of the recent estimators are based on the Multiple Signal Classification (MUSIC) estimator which has been originally developed for narrow band incoherent sources environment problems. Theoretical study of the MUSIC estimator shows that it possess good asymptotic variance, but with high resolution threshold and driven bias. In the last few years, an attention has been given to reduce the estimation bias and/or resolution threshold. An improvement of both, in some works, comes on the account of the variance of the estimated angles. Recently, two MUSIC like estimators - the weighted norm MUSIC and D_r estimators have been proposed [1]. The analytical and simulation studies of these estimators in narrow band incoherent sources environment show that they have the same asymptotic variance of the MUSIC but with lower bias and resolution limit. In the environment of coherent sources, one has to expect a

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performance much less than that has been estimated. The most widely used techniques for alleviating this problem are the spatially smoothing [2,3], and focusing techniques [4,5]. Normally, one of these techniques is combined with a selected DOA's estimator to resolve the signal impinging on an array of sensors.

This paper presents a comparison between the MUSIC in conjunction with the Rotational Signal Subspace (RSS) focusing techniques [5], and the D_r estimators using a uniform linear array. The results show also that a better performance than both can be attained when the D_r estimator is combined with the RSS technique. Different array geometries can also be employed using the transformation techniques as described in [6,7].

II. Model and Basic Assumptions

Let \mathbf{R} is the sample covariance matrix based on N independent snapshots of the received signal from uniform linear array of L elements. The geometry of the array is described by the steering vector $\mathbf{v}(\vartheta)$ in the elevation as

$$\mathbf{v}(\vartheta) = [1, e^{i2\pi f D \cos(\vartheta)/c}, \dots, e^{i2\pi f D(L-1) \cos(\vartheta)/c}] \quad (1)$$

where D is the interelement spacing, c is the wave velocity in space, f is the frequency of the received signal, and ϑ is the angle between the direction of arrival and the array axis. Assume that λ_i and \mathbf{e}_i are the i^{th} eigenvalue and normalized eigenvector of \mathbf{R} . If the first d eigenvalues are distinct and greater than the variance of an additive white Gaussian noise σ_n^2 ; i.e.

$$\lambda_1 > \lambda_2 > \dots > \lambda_d > \lambda_{d+1} = \dots = \lambda_L = \sigma_n^2, \quad (2)$$

the signal subspace S_s and noise (null) subspace S_n are defined as

$$\begin{aligned} S_s &= \text{span}\{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_d\} \\ S_n &= \text{span}\{\mathbf{e}_{d+1}, \dots, \mathbf{e}_L\} \end{aligned} \quad (3)$$

In the following, the definitions

$$\begin{aligned} E_s &= [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_d], \\ E_n &= [\mathbf{e}_{d+1}, \dots, \mathbf{e}_L], \\ P_s &= E_s E_s^H, \text{ and} \\ P_n &= E_n E_n^H \end{aligned} \quad (4)$$

are used in the expression of the MUSIC and D_r estimators, where $[\ast]^H$ denotes the hermitance transpose of the vector $[\ast]$. The eigenanalysis of \mathbf{R} is carried at K discrete frequencies uniformly distributed over the bandwidth of interest.

The MUSIC Estimator

The MUSIC estimator is defined, for wide band sources, as [8]

$$P(\vartheta) = \frac{1}{\frac{1}{K} \sum_{k=1}^K \|(I - E_s E_s^H) \nu(\vartheta)\|_{f_k}^2} \quad (5)$$

The D_r Estimator

The D_r estimator, a class of the MUSIC, is formulated as a minimization of the Euclidean distance between two vectors in the signal subspace [1]. Let the set E_s is decomposed into $E_s^{(1)}$ and $E_s^{(2)}$, and define

$$\begin{aligned} f^{(1)}(\vartheta) &= (I - E_s^{(1)} E_s^{(1)H}) \nu(\vartheta) \\ f^{(2)}(\vartheta) &= E_s^{(2)} E_s^{(2)H} \nu(\vartheta) \end{aligned} \quad (6)$$

The D_r estimator is defined by

$$P^{D_r}(\vartheta) = \frac{1}{\frac{1}{K} \sum_{k=1}^K \min. [\|f^{(1)}(\vartheta)\|^{r(\vartheta)} - \|f^{(2)}(\vartheta)\|^{r(\vartheta)}]_{f_k}} \quad (7)$$

where

$$r(\vartheta) = \frac{1}{\log \|f^{(2)}(\vartheta)\|} \quad (8)$$

The RSS Estimator

The general technique of focusing is based on averaging the covariance matrices at the selected K discrete frequencies in the form

$$\bar{R} = \sum_{k=1}^K T_k R_k T_k^H \quad (9)$$

where T_k and R_k are the focusing and covariance matrices at the frequency f_k , respectively. The eigenvectors of the steered covariance matrix is used for estimating the directions of radio activities. The form of the estimator is similar to that of the MUSIC for narrow band sources. The T_k is defined as a rotational matrix of the steering vector $\nu(\vartheta)$ at f_k to that at the center frequency f_o . The detail of T_k generation is given in [5].

III. Simulation Results

Let the number of omni-directional sensors of a uniform linear array is 7 with interelements spacing is 0.5 the wavelength λ of the center frequency f_o . Consider a problem of two closely spaced equipower wideband coherent sources with band width $0.125 f_o$. The sources direction are defined by the center angle ϑ_o between the sources measured from the array axis, and the angle of separation Δ normalized to the BeamWidth (BW) of the array. Let Δ equals 0.2 BW, and select the width of the focusing domain to 1.25Δ ; the optimum focusing width as proposed in [7]. Figure 1 shows both the bias and variance, normalized to BW, of the estimated directions versus SNR in dB using the RSS, D_r , and D_r -RSS estimators in the two cases of $\vartheta_o = 90^\circ$ and 60° . Figures 1.a and b, for $\vartheta_o = 90^\circ$, show that the resolution threshold of the RSS is 10dB and higher than that of the D_r , but with lower bias and variance. This result gives rise to an idea of combining the D_r with RSS estimators for attaining the lower resolution threshold of the D_r , with a lower level of both the bias and variance of the RSS. In this approach, which is named D_r -RSS, the D_r is applied to a single focused covariance matrix of the RSS. The dash-dot curves in Figs. 1.a and b show that the D_r -RSS has a performance much better than that of the D_r and RSS. In the above problem with $\vartheta_o = 60^\circ$, the performance of the three estimators are less than those in case of $\vartheta_o = 90^\circ$; compare Figs. 1.a with 1.c, and 1.b with 1.d. Figures 1.c and d show also that the performance of the D_r -RSS is better than the others.

The main difficulties in the application of the RSS estimator are the preliminary estimation of both the ϑ_o and the width of the focusing domain. In the above results, the angle ϑ_o is chosen in the direction of a single peak in the MUSIC estimated spectrum, while the focusing width is increased in steps of length $.01BW$ until minimum levels of both the bias and variance are attained. Indeed, this is a complicated process to be applied in practice. Therefore the following process, named D_r /RSS, is suggested to simplify the application of the RSS. Consider the above problem with $\vartheta_o = 90^\circ$, and SNR=5dB. The estimated spectrum of the MUSIC estimator (Fig.2.a) shows an unresolved spectrum with a single peak in the direction of $\vartheta_o = 90^\circ$. The arrows in the Figure mark the real directions of the sources. This single peak may be interpreted as a single source in the peak direction, or an unknown number of coherent sources in unknown directions. Since the D_r estimator has a low resolution threshold, it can be used to resolve this peak, and hence preliminary estimate the sources location. Fig.2.b shows that the estimated spectrum of the D_r comprises two distinct peaks in the directions of $\vartheta_o = 90^\circ$ and $\vartheta_o = 90^\circ$. The next step is applying the RSS using two groups of focusing in ϑ_1 and ϑ_2 directions with a domain width of focusing for each is $\leq |\vartheta_1 - \vartheta_2|$. Figure 2.c shows that the peak of

the spectrum become very close to the real directions. For further refinement, the RSS can be applied once again in the new directions, but with reduced domain of focusing.

IV. Conclusion

The D_r estimator, originally developed for narrow band incoherent sources, has a low resolution threshold when applied in the environment of wide band coherent sources. This feature can be used to circumvent the difficulties in the application of the Rotational Signal Subspace (RSS) focusing technique. These difficulties are the preliminary estimation of both the sources direction and the domain width of focusing. The numerical results show that the conjunction of the D_r and RSS estimators in a process named D_r /RSS may yields a much better performance than each.

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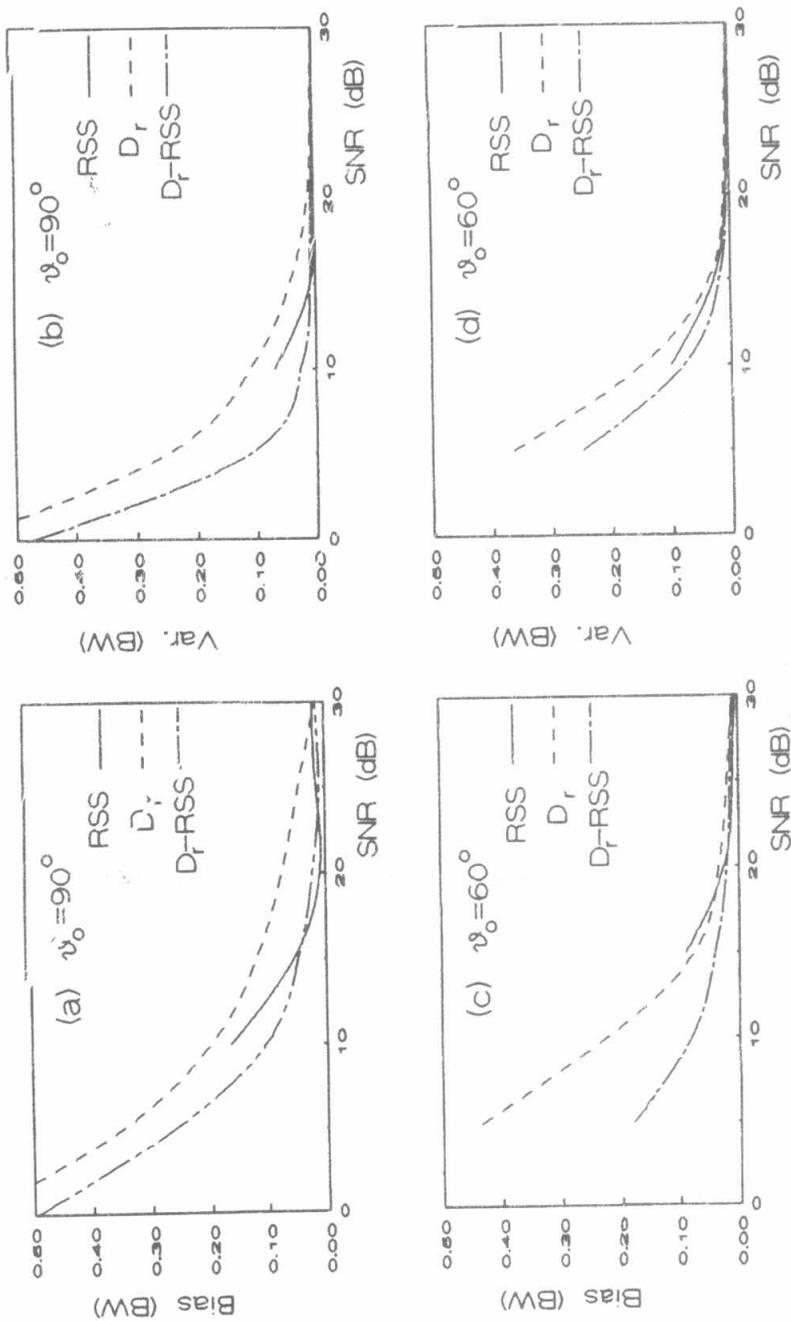


Fig.1 The bias and variance of the estimated directions vs. SNR using RSS, D_r , and D_r -RSS. (a) and (b) $\psi_0 = 90^\circ$, (c) and (d) $\psi_0 = 60^\circ$

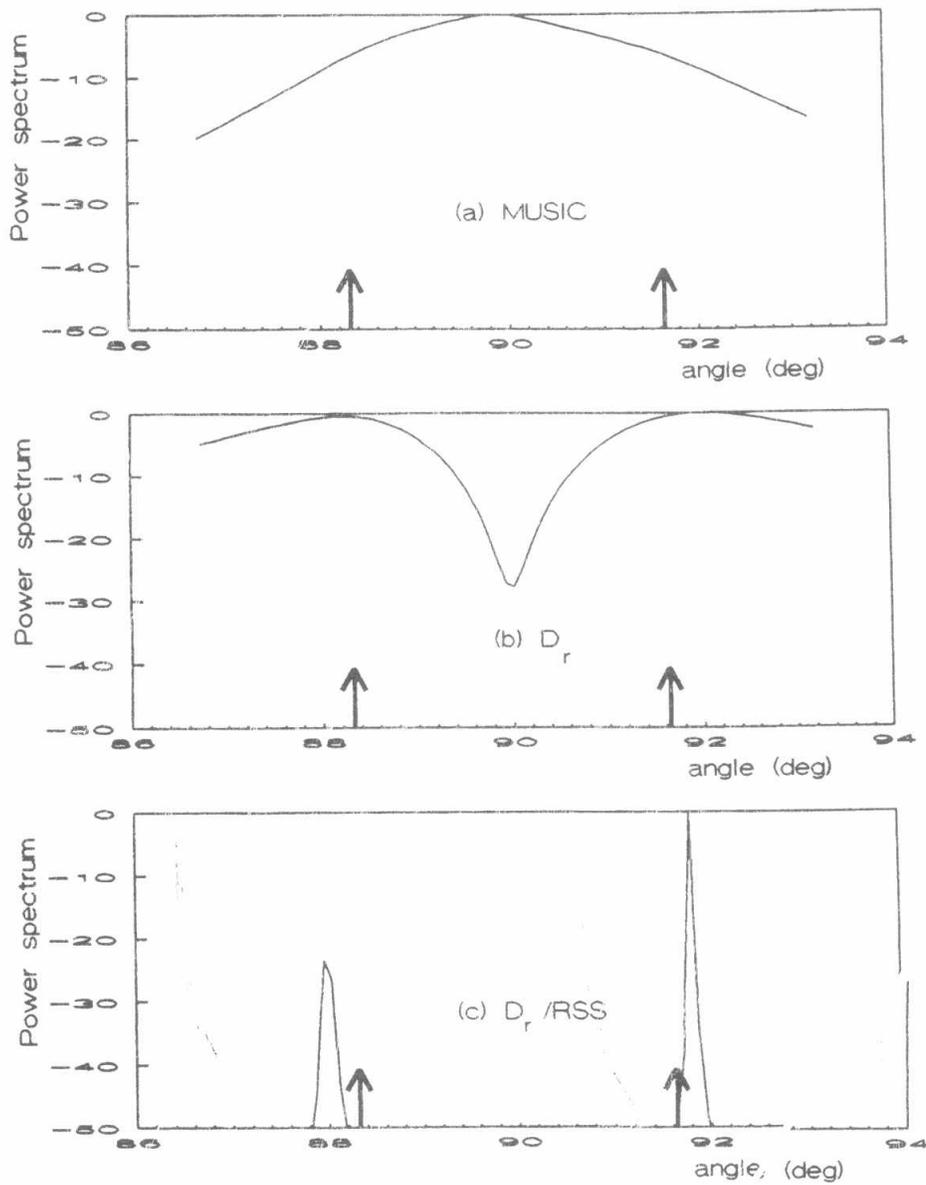


Fig.2 The estimated power spectrum of two coherent sources.
(a) MUSIC (b) D_r (c) D_r/RSS