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Performance of WPDM in AWGN and Timing Error for Different Types of Wavelet packet Basis Functions

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Abstract - *Wavelet packet division multiplexing (WPDM) is a high capacity, flexible and robust orthogonal multiplexing technique in which wavelet packet basis functions are chosen as the coding waveforms. This paper simulates different types of signals of wavelet packet division multiplexing transmitter-receiver system operating in additive white Gaussian noise (AWGN). The interference caused by timing offset in transmission is also examined by Monte Carlo simulation method to indicate that wavelet packets appear to be promising waveforms in multiple signal transmission techniques.*

1. Introduction

In digital communication systems multiplexing refers to the transmission of a number of independent messages over a common channel. Orthogonal waveform coding as remained the most effective technique for multiplexing because the data symbols of each user can be recovered from the transmitted signal using a simple correlator [1]. The commonly used time division multiplexing (TDM), frequency division multiplexing (FDM), the orthogonal code division multiplexing (CDM)[2] are all examples employing orthogonal waveform coding in multiplexing. An emerging alternative to these conventional orthogonal multiplexing schemes is the wavelet packet division multiplexing (WPDM) [4], which exploits the self-and mutual-orthogonality properties of wavelet packet basis functions for multiplexing purposes. In contrast to the conventional TDM, FDM and CDM schemes, the waveforms used in WPDM overlap in *both* time and frequency, while the wavelet packet structure insures that the waveforms are orthogonal. This provides a substantial increase in capacity over TDM and FDM whilst the close relationships with transmultiplexers provide particularly simple transmitter and receiver structure. In this paper we use Monte Carlo simulation method to examine the performance of different types of WPDM signals under the effect of AWGN, and the interference caused by timing offset in transmission, to demonstrate that the degree of freedom implicit within the wavelet packet frame work can be exploited to design sets of waveforms which are robust to common imperfections in physical implementations.

2. Wavelet and wavelet Packet

The theory of wavelet and wavelet packet is by now a well-developed subject. However, in order to keeps the paper self-contained we briefly summarize the concept of wavelet packets. We will start from multiresolution analysis. A multiresolution analysis (MRA) consists of a collection of embedded subspaces $V_{\ell 1}$

$$V_{\ell 1} \subset \dots \subset V_{21} \subset V_{11} \subset V_{01} \qquad (1)$$

Where $\ell = 0,1,2,\ldots$ each subspace $V_{\ell 1}$ has orthonormal basis $\{\phi_{\ell 1}(t - nT_\ell)\}$ generated by translations of a single function $\phi_{\ell 1}(t)$. Furthermore, the basis functions for different subspaces fig.2 can be obtained from each other by dyadic dilation; i.e.,

$$\phi_{\ell 1}(t) = 2^{\ell/2}\phi_{01}(2^{-\ell}t) \qquad (2)$$

and $T_\ell = 2^\ell T_o$. Science $V_{\ell 1} \subset V_{\ell-1,1}$ we can write $\phi_{\ell 1}(t)$ as a linear combination of the orthogonal basis functions of $V_{\ell-1,1}$ i.e.:

$$\phi_{\ell 1}(t) = \sum_n h[n]\phi_{\ell-1,1}(t - nT_{\ell-1}) \qquad (3)$$

Where

$$h[n] = \langle \phi_{\ell-1,1}(t - nT_{\ell-1}), \phi_{\ell 1}(t) \rangle \qquad (4)$$

and $\langle .,. \rangle$ denotes the inner product, given such a sequence h[n] we can find a sequence g[n] satisfying

$$\sum_n g[n]h[n-2m] = 0 \qquad (5)$$

Such that the function

$$\phi_{\ell 2}(t) = \sum_n g[n]\phi_{\ell-1,1}(t - nT_{\ell-1}) \qquad (6)$$

The sequence g[n] can be chosen to be a reversed, and shifted version of h[n]

$$g[n] = (-1)^n h[1-n] \qquad (7)$$

Resulting basis functions $\phi_{\ell 1}(t)$ and $\phi_{\ell 2}(t)$ are called the scaling function and wavelet at scale ℓ, respectively. In an analogous way to Eqs (3) and (6), we can write

$$\phi_{\ell+1,2m-1}(t) = \sum_n h[n]\phi_{\ell m}(t - nT_\ell) \qquad (8)$$

$$\phi_{\ell+1,2m}(t) = \sum_n g[n]\phi_{\ell m}(t - nT_\ell) \qquad (9)$$

Where h[n] and g[n] are the coefficients of **L.P.F** and **H.P.F** and here the first subscript denotes the "level" in the tree, and the second denotes the position of a node in a given level .We can grow the tree in any desired fashion , the function $\phi_{\ell m}(t)$ at the leafs or terminals of a tree structure provide a set of "wavelet packet basis functions" or simply a wavelet packet, as exemplified in fig.1. The corresponding subband structure in the frequency domain is shown in fig.2.

3. WPDM Transmitter and Receiver: -

In our application of wavelet packets to multiple signal transmission, we consider a TDM system. The system that we propose here seeks the representation of the binary symbols "1"' and "-1" by $\phi_{\ell m}(t)$ and $-\phi_{\ell m}(t)$[4], respectively, so that the waveform coded composite TDM sequence is given by:

$$S_{\ell m}(t) = \sqrt{E_o}\sum_n \sigma_{\ell m}[n]\phi_{\ell m}(t - nT_\ell) \qquad (10)$$

Here E_o is the energy of one symbol and the combined sequence forms a composite sequence of binary symbols $\sigma_{\ell m}[n] = \pm 1$. The optimal receiver in additive white gaussian noise is the well-known correlator followed by sampler. In practice, the scaling function $\phi_{01}(t)$ [3] will usually be chosen to have finite duration, so that the constituent functions $\phi_{\ell m}(t)$ are also of finite duration [4] (and the filters h[n] and g[n] are FIR filters). The configuration of this multiple signal transmission system is shown in the following Fig.4. In which the tree structure fig.1 has been chosen to have M terminals. At the transmitter out put we have a composite signal given by:

$$S(t) = \sqrt{E_o}\sum_{\ell\in l, m\in M}\sum_n \sigma_{\ell m}[n]\phi_{\ell m}(t - nT_\ell) \qquad (11)$$

The reception of such a composite signal can be carried out using bank of correlators as shown in fig.4, It is noted that since the WPDM system transmits orthogonal antipodal waveforms to represent the binary data, its performance in the AWGN channel is identical to any other transmission system, which uses such waveforms. But WPDM is more efficient in the bandwidth usage, since no guard time is necessary. And the constituent functions used in WPDM can overlap with one another in both Time and frequency domains, we can expect that the multiplexing is more efficient in Bandwidth usage than conventional FDM systems. Signal transmission schemes based on synchronous correlators are vulnerable to errors caused by timing discrepancy. We make the following assumption, although the timing discrepancy is, in general, a random variable, we assume that it is changing slowly so over the transmission of a group of symbols, it can be a constant error. Let "Δ" denote the timing discrepancy between transmitter and receiver. In attempt to recover the transmitted signal. The output of the correlator in the receiver is given by:

$$\hat{\sigma}_{01}[n] = \sum_k \sigma_{01}[k] \int \phi_{01}(t - kT_o)\phi_{01}(t - nT_o + \Delta)dt$$
$$= \sum_k \sigma_{01}[k] R_\phi(kT_o - nT_o + \Delta) \qquad (12)$$

Where $R_\phi(\tau)$ is the autocorrelation function of $\phi_{01}(t)$. Continuing the derivation to find the approximate probability of error due to timing discrepancy and AWGN we get the following relation [4].

$$P_{lm}(\varepsilon) = \frac{1}{2} erfc\left(\frac{1}{\sqrt{N_o / E_o + 2\varsigma^2_{\ell m}}}\right) \qquad (13)$$

Where P_{lm} is the probability of error, N_o/E_o is the signal to noise ratio, $\varsigma_{\ell m}$ denote the standard deviation of the signal due to discrepancy. We now introduce a Monte Carlo simulation done with Visual Basic program, to study the performance of WPDM under AWGN and timing error for different types of wavelet packet basis functions (Har,Db6,Sym6) . We used 4800 bit distributed on four users, and a(Haar,Db6,Sym6) basis function for waveform shaping. The program goes as follows, first we generate random bits for each user then the wavelet packet tree is constructed and the waveform shaping is presented using db6. AWGN is added to the multiplexed signals and the constant timing discrepancy $\Delta = 0.15 \ T_o$ as an example is applied on the transmitted signal, then the received bits are calculated and the bit error rate is calculated, at that point we increase the noise level and recalculate the bit error rate and so on . then we repeat all steps for Hara and Sym6. The bit error rate of each case is then plotted versus signal to noise ratio fig.5 and the theoretical error rates given by (15) are also plotted as reference(solid line)fig.5, however there is a discrepancy between the theoretical evaluation of performance and that from simulation because the theoretical probability of error due to timing Error is derived from approximate relation [4].

4. Conclusion

In this paper, a new scheme for transmitting multiple signals wavelet packet (WPDM) has been proposed, which offers capacity improvement over the commonly used FDM and TDM schemes. Since the specific choice of wavelet is application-dependent, no analytical design rules exist to pick a good wavelet for a given application, we introduce a Monte Carlo algorithm to study the performance of WPDM under AWGN and timing error for different types of wavelet packet basis functions

(Har,Db6,Sym6). We note that the performance of WPDM using "Db6" is almost identical to that obtained by "Sym6", and being slightly superior to the use of Haar function. Because the Symlets are nearly symmetrical wavelets proposed by Daubechies as a modification to the "Db" family [3], and the discontinuity of "Haar" function, which causes the spectrum, does not decay rapidly. One of the main disadvantages of the Monte Carlo method is time consuming and this can be overcome by using the new fast commercial PC's.

References

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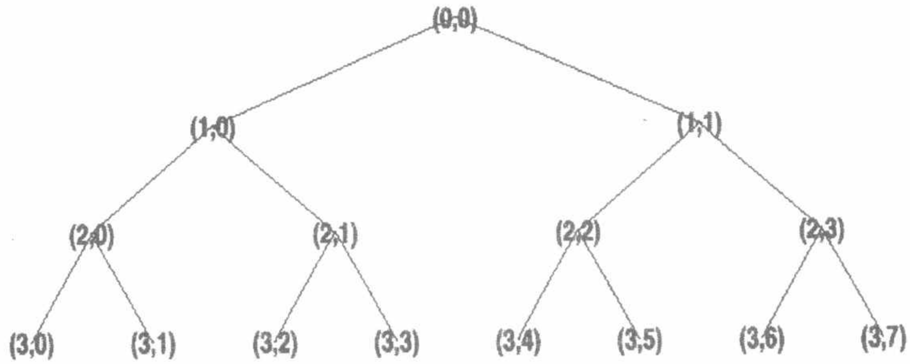

Fig.1 Typical WPDM Tree Structure

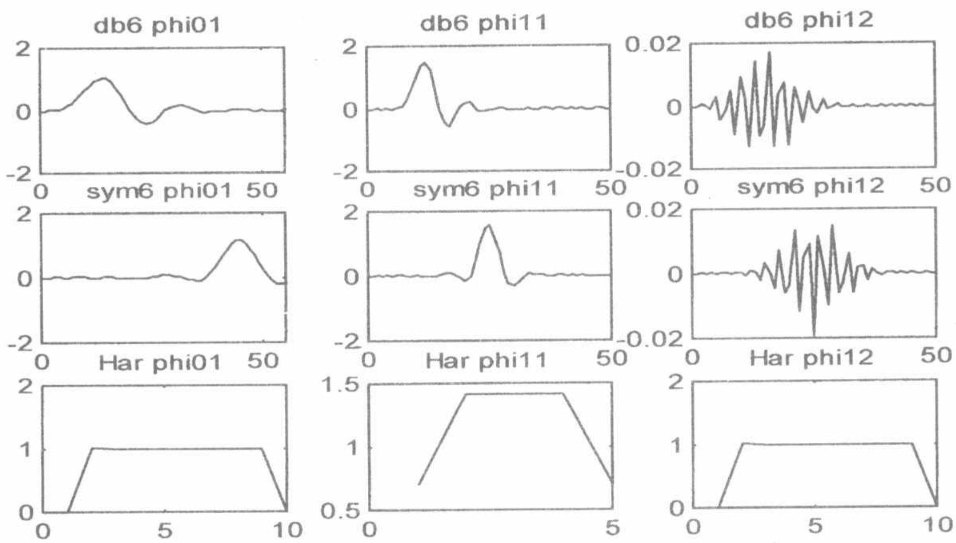

Fig.2. Different Wavelet Packet Basis Functions

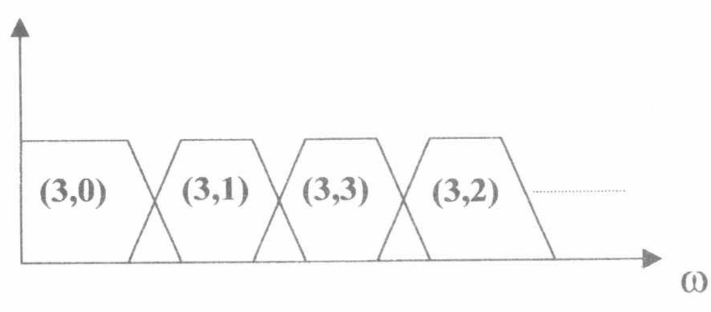

Fig.3 Subband structure of the tree

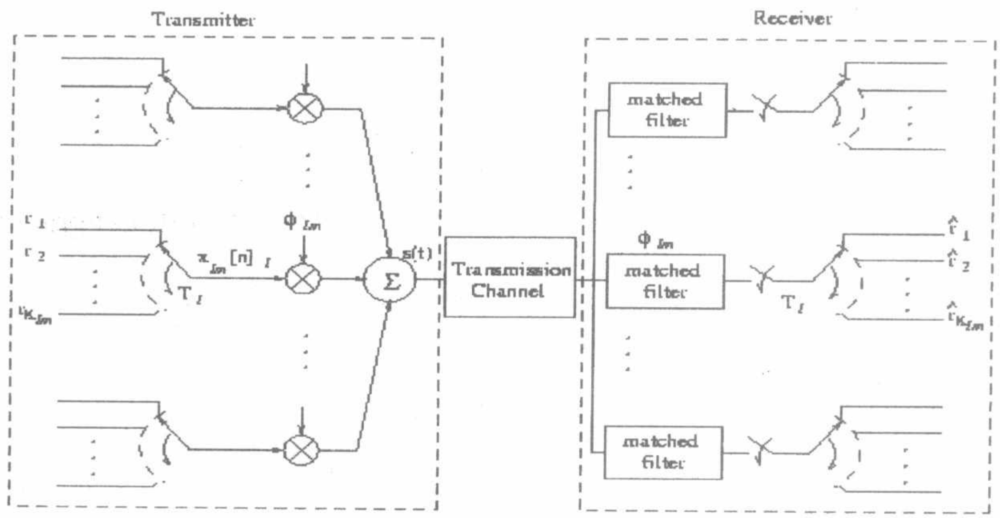

Fig.4 WPDM Transmitter and Receiver

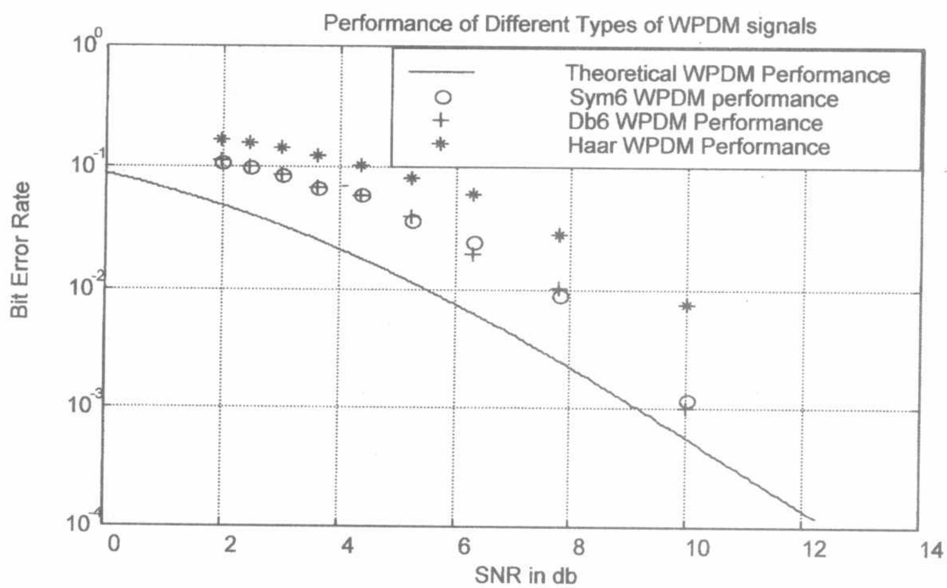

Fig. 5. Performance of Different Types of WPDM Signals